Contents

Preface vii

Chapter 1. On the $\Psi-$ Conditional Asymptotic Stability of Nonlinear Lyapunov Matrix Differential Equations 1

1.1. Introduction 1
1.2. Preliminaries 2
1.3. $\Psi-$ conditional asymptotic stability of linear matrix differential equations (1.4) and (1.9) 8
1.4. $\Psi-$ conditional asymptotic stability of linear Lyapunov matrix differential equations 14
1.5. $\Psi-$ conditional asymptotic stability of non-linear Lyapunov matrix differential equations 22

Bibliography 33

Chapter 2. Differential equations of upper order 35

2.1. Boundary value problems on half-line 35
2.1.1. Introduction 35
2.1.2. Asymptotic behavior of solutions to damped nonlinear oscillators 36
2.2. Boundary value problems on the real line 42
2.2.1. Existence of coalescent solutions to a nonlinear second order ODE 42
2.2.2. Solutions to second order ODEs having finite limits at $\pm\infty$ 49
2.2.3. Coalescent solutions via Lyapunov function 62

Bibliography 69

Chapter 3. Numerical simulations and some applications in cortical bone behavior 73

3.1. Introduction 73

3.2. Human cortical bone ... 73
3.2.1. The bony architecture ... 74
3.3. SiNuPrOs model: homogenized properties ... 77
3.3.1. Piezoelectric framework ... 77
3.3.2. Short description of the AHM development ... 79
3.3.3. Study of the convergence ... 90
3.3.4. Determination of the physical properties at each level ... 98
3.4. Results and discussions ... 100
3.4.1. Parametric study: effect of the microstructure on the macroscopic properties ... 100
3.4.2. Effect of the orientation of collagen fibers ... 104
3.5. Conclusions and perspectives ... 109

Bibliography ... 113

Chapter 4. On the variable exponent spaces and their applications ... 115
4.1. Introduction: applicability and history notes ... 115
4.2. Preliminaries on the spaces with variable exponent ... 119
4.3. Generalized problems treated in the framework of the anisotropic spaces with variable exponent ... 129

Bibliography ... 143

Chapter 5. On some elliptic higher order equations ... 147
5.1. Introduction ... 147
5.2. Assumptions and Notations ... 150
5.3. Some Useful Results ... 150
5.4. Main Results ... 152
5.4.1. The general case ... 152
5.4.2. The case $m = 2$. Two equations arising in plate theory ... 156

Bibliography ... 163

Chapter 6. Non-commutative Inequalities in Operator Algebras ... 165
6.1. A "link" between inequalities and non-commutativity ... 165
6.2. Completely positive inner products and a non-commutative Schwarz inequality ... 168
6.3. Annex ... 174

Bibliography 179

Chapter 7. Lagrangians adapted to foliations and nonholonomic structures 181
7.1. Introduction 181
7.2. Scalar products, Lagrangians and Finslerians on vector bundles 183
7.3. Lagrangians, epimorphisms and submersions 187
7.3.1. Epimorphisms over the same base 187
7.3.2. Epimorphisms over different bases 195
7.4. Lagrangians and foliations 197
7.4.1. Lagrangians on foliations and geodesics 203
7.4.2. Positive definite transverse Lagrangians and foliations 206
7.5. Foliated vector bundles and transverse jet bundles 209
7.5.1. Proof of the main results related to higher transverse jet bundles 211
7.6. Basic notions and constructions related to the higher order transverse bundles of a foliation 213
7.6.1. The main results related to higher order transverse bundles 221
7.7. Linear, affine and nonlinear constraints and Lagrangians . 224
7.7.1. Linear and affine constraints 224
7.7.2. Non-linear constraints 227
7.7.3. The Lagrangian dynamics for linear, affine and nonlinear constraint systems 235
7.7.4. Examples of nonholonomic constraints 240
7.7.5. A discrete approach to nonholonomic constraints . 246

Bibliography 259

Chapter 8. Geometrical Methods for the Study of Dynamical Systems. Symmetries and Conservation Laws 263
8.1. Introduction 263
8.2. Basic Tools 264
8.3. The Symplectic Formalism 266
8.4. Symmetries, Pseudosymmetries and Conservation Laws for Presymplectic Systems 270
8.4.1. Introduction 270

8.4.2. The Presymplectic Formalism 270
8.4.3. A Study of Some Biological and Ecological Dynamical Systems 275
8.5. Symmetries, Psudosymmetries and Conservation Laws in Lagrangian and Hamiltonian k-Symplectic Formalism 278
8.5.1. Introduction 278
8.5.2. Geometric Framework of Günther k-Symplectic Formalism 280
8.5.3. Symmetries in k-Symplectic Field Theories 291
8.5.4. Generalized Results and Applications to k-Symplectic Hamiltonian and Lagrangian Systems . 298

Bibliography 307

Chapter 9. A Geometrical Study of Systems of Ordinary Differential Equations (SODEs) and Second Order Partial Differential Equations (SOPDEs) 311
9.1. On the 2-Semispray of Nonlinear Connections in 2-Tangent Bundle Geometry 312
9.1.1. The 2-semispray of a nonlinear connection 312
9.1.2. The 2-semispray of a nonlinear connection in a Lagrange space of second order 320
9.2. On the k-Semispray of Nonlinear Connections in k-Tangent Bundle Geometry 323
9.2.1. The k-Semispray of a Nonlinear Connection 323
9.2.2. The k-Semispray of a Nonlinear Connection in a Lagrange Space of Order k 332
9.3. Second order partial differential equations (SOPDEs) and nonlinear connections on the tangent bundle of k^1-velocities of a manifold 335
9.3.1. Introduction 335
9.3.2. SOPDEs and nonlinear connections on $T^1_k M$ 345

Bibliography 351

Chapter 10. Geometric and Numerical Methods in the Characterization of Dynamical Systems 355
10.1. Introduction 355

10.2. A Numerical Study of Conservation Laws for the Hamiltonian Dynamical System Associated to the Isotropic Harmonic Oscillator 355
10.2.1. Problem statement 355
10.2.2. Classical Results 357
10.2.3. Numerical Study 364
10.2.4. Conclusions 380
10.3. Geometrical Methods and Numerical Computations for the Study of Mathematical Models of the Multi-Species Interactions 380
10.3.1. Introduction 380
10.3.2. The prey-predator $2D$ Lotka-Volterra system 382
10.3.3. The Bailey model for the evolution of epidemics 388
10.3.4. The $3D$ Lotka-Volterra system 392
10.3.5. Conclusions 399

Bibliography 401

Chapter 11. Computational approach for the mixing flow dynamical system. The parameter influence on the trajectory behavior 405
11.1. Introduction. The mixing mathematical context 406
11.2. Recent results 408
11.3. The perturbed mixing model. Graphical analysis 409
11.4. Conclusions. Remarks 412

Bibliography 415