Daniela DANCIU

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QUALITATIVE ANALYSIS AND CONTROL OF NONLINEAR SYSTEMS.

COMPUTATIONAL MODELING AND STABILIZATION OF DISTRIBUTED PARAMETER SYSTEMS

Recent Contributions

EDITURA UNIVERSITARIA
Craiova, 2021

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Descrierea CIP a Bibliotecii Naţionale a României
DANCIU, DANIELA
Qualitative analysis and control of nonlinear systems : computational modeling and stabilization of distributed parameter systems : recent contributions / Daniela Danciu. - Craiova : Universitaria, 2021
Conţine bibliografie
ISBN 978-606-14-1753-7

681.5

PREFACE

The purpose of this book is to present the author's recent contributions in the field of dynamical systems, and their applications to modeling, control and/or stabilization. The book is a presentation of the author's endeavors, being a result of the effort *to pass beyond*. To pass beyond the starting point in a research career – the Ph.D. thesis – is not an easy task. The author tried this in various ways, stimulated by the external and career requirements, but acting also in virtue of personal views and principles. Written on the basis of the author's habilitation thesis, this book is suitable for PhD students as well as for researchers with background and interests in the theory and applications of *Control Engineering*, *Applied Mathematics*, *Computational Mathematics* and *Artificial Intelligence* (AI). The book is organized into three distinct parts. Each part begins with an abstract and an introductory chapter and, at the same time, represents a research direction pursued by the author through the topics of the research projects she participated in as a director, principal investigator or a member, as well as topics of individual research interest.

Part I consists of three chapters. Chapter 1 briefly introduces the framework of the *Qualitative Analysis for Systems with Multiple Equilibria* (SME) as well as the class of *Recurrent Neural Networks* (RNNs). Chapter 2 gives results regarding conditions to be fulfilled by RNNs as SME in order to possess the qualitative behavior desirable for achieving the functional tasks they are designed for – the gradient-like behavior. The results and applications of Chapter 3 extend the study to the case of RNNs with multiple nonlinearities and lumped time-delays.

Part II considers an important research direction for the control engineering field: the absolute stability property for nonlinear dynamical systems. The collaborative results of this part were motivated by their applications to a challenging control problem – the *Pilot-Induced-Oscillations* (PIO) for aircrafts – i.e., the critical or unstable cases due to the actuators saturation nonlinearities in interaction with the pilot's sudden and unpredictable manoeuvrers. Chapter 4 presents the state-of-the-art for the absolute stability field and a short introduction with the classification of the PIO phenomena. Chapter 5 introduces two results for absolute stability of SISO (single input - single output) and MIMO (multiple input - multiple output) systems with both sector and slope restricted nonlinearities. Chapter 6 contains a selection of illustrative examples for the application of the previous introduced results of absolute stability.

Part III is devoted to the field of *Distributed Parameter Systems* (DPS). Having a multi- and interdisciplinary research character, it focuses on the computational modeling and control of *Distributed Parameter Control Systems* (DPCS). Chapter 7 introduces a well-structured computational procedure based on a “convergent Method of Lines” combined with certain AI devices in order to model DPS described by hyperbolic partial differential equations (hPDEs) with non-standard boundary conditions (BCs). Chapter 8 extends this procedure to the case of parabolic PDEs (pPDEs). Before introducing some illustrative applications, in Chapter 9 we consider the boundary stabilization for a system with distributed parameters; the behavior of this closed loop system will be further investigated in the next chapter *via* computational modeling. There are multiple applications of the procedure for computational modeling and, thus, the behavior evaluation of those DPCS arising from real-world problems belonging to various areas such as: contact mechanics, oil extraction industry, biotechnology, cogeneration (combined electricity and heat generation), water hammer phenomenon in hydraulics, cranes with flexible cable (used, for instance, in transportation, manufacturing, construction, offshore platforms etc.) and so on. Chapter 10 presents a selection of five such applications of the procedure for DPCS within the aforementioned areas. From the mathematical point of view, the proposed procedure is suitable for mixed initial boundary value problems for PDEs with non-standard boundary conditions. Each such problem has its own specificity induced by both the PDEs model and the non-standard BCs. Each of the considered applications is accompanied by simulated experiments, 2D and 3D graphical representations, as well as results interpretations from both points of view of transient and asymptotic behavior; furthermore, discussions concerning the computational efficiency of the procedure on each case are provided.

Since this book has multiple faces, it is not without interest to give to the potential reader what is unifying these various research directions above mentioned. First, it is the concept of *dynamical system*, in the form arising from the ideas of Henri Poincaré and George David Birkhoff: an evolution along the time, generated (“ignited”) by the initial conditions. In turn, the initial conditions incorporate the effect of the short duration perturbations on a basic evolution of the system. This point of view belongs to Aleksandr M. Lyapunov; Rudolph E. Kalman put it explicitly: the initial conditions “integrate” system’s pre-history. Going further, Nikolai G. Četaev pointed out that real-world dynamical systems are subject to a quasi-permanent small amplitude perturbation field. Therefore, stability (in the sense of Lyapunov) becomes crucial, being (Četaev dixit) a part of the Nature laws. Since *Dynamics* is the science of the real (and effectively observable/measurable) equilibria and motions of the material systems, these equilibria and motions are observable/measurable provided they are stable. Otherwise, the aforementioned perturbation field will “obscure”, if not “destroy”, them. This general property of stability should be viewed as completing the necessary criteria for model validation, criteria which are critical in Physics, Engineering, Biology, “soft” sciences (e.g. Machine learning, Artificial Intelligence) – disciplines which are found within this book under the methodological “umbrella” described above.

Craiova, 2021 *Daniela Danciu*

PART I

QUALITATIVE ANALYSIS FOR DYNAMICAL NEURAL NETWORKS AS SYSTEMS WITH SEVERAL EQUILIBRIA

ABSTRACT

Dynamical systems with several equilibria occur in such fields of science and technology as electrical machines, chemical reactions, economics, biology and, last but not least, neural networks. For systems with several equilibria the usual local concepts of stability are not sufficient for an adequate description. The so-called "global phase portrait" may contain both stable and unstable equilibria: each of them may be characterized separately since stability is a local concept dealing with a specific trajectory. Nevertheless, global concepts are also required for a better system description and this is particularly true for the case of the neural networks.

The Recurrent Neural Networks (RNNs) may be viewed as interconnections of simple computing elements whose computational capability is increased by interconnection ("emergent collective capacities" – to cite J.J. Hopfield [95]). This is due to the nonlinear characteristics leading to the existence of several stable equilibria. The network achieves its computing goal if no self-sustained oscillations are present and it always achieves some steady-state (equilibrium) among a finite (while large) number of such states. The contributions presented in this part of the manuscript are structured in the following chapters.

Chapter 1 introduces the basics for recurrent neural networks as nonlinear systems with several nonlinearities and/or equilibria and the main "tools" for investigating their qualitative behavior. After a short introduction which presents the general context emphasizing the importance of the studies presented in this part, the chapter continues with the main concepts and results of the *Qualitative Theory of Systems with Several Equilibria* (QTSSE) and the description of the most used and studied recurrent neural networks which are also studied in this part of the manuscript. Some concluding remarks stress the desirable behaviors for RNNs from the point of view of the theory of systems with several equilibria.

Chapter 2 presents some contributions concerning the qualitative behaviors of RNNs as dynamical systems with several equilibria and nonlinearities. In the first section the static and dynamic analysis of KWTA networks is performed within the framework of QTSSE, and conditions for the gradient-like behavior of the KWTA neural networks are given. In the second section we consider a general model for RNNs with sector and slope restricted nonlinearities within the framework of the *Absolute stability theory*; the results are then particularized to the case of analog Hopfied circuits and further discussed for other RNNs.

Chapter 3 takes into consideration the dynamics of RNNs with multiple nonlinearities and/or several equilibria which are affected by time-delays due to the signal propagation at the synapses level or to the reacting delays in the case of the artificial neural networks. Due to their undesirable effects on the systems dynamics, such as oscillations or instabilities, beginning with the years of 90-ies of the 20th century, these phenomena have been included in the NNs' mathematical models and various qualitative behavior studies have been considered. Time-delay systems (TDS) are modeled by functional differential equations and the approaches and tools we have used for their qualitative analysis include: the *Lyapunov-Krasovskii functional*, the *frequency domain method* by V.M. Popov as well as the *comparison principles* that lead to differential and integral inequalities. This chapter consists of the following contributions: the global asymptotic stability for time-delay Hopfield networks (a Lyapunov-type approach), synchronization problems for time-delay RNNs (by using the both Lyapunov and Popov approaches), robust exponential stability for time-delay CNNs, the Popov-like results using comparison for RNNs as systems with multiple equilibria and time-delays, an extension of the LaSalle-like theory for systems with multiple equilibria and time-delays.

The results presented in Part I were introduced in the following publications:

1. D. Danciu (2011). Bio-inspired Systems. Several Equilibria. Qualitative Behavior, *Proc. 11th Int. Work-Conference on Artificial Neural Networks*, IWANN'2011, Lect. Notes on Comp. Sci., Springer, vol. 6692, pp. 573–580.
2. D. Danciu, V. Răsvan (2011). Systems with Slope Restricted Nonlinearities and Neural Networks Dynamics, *Proc. 11th Int. Work-Conference on Artificial Neural Networks*, IWANN'2011, Lect. Notes Comp., Springer, vol. 6692, pp. 565-572.
3. D. Danciu (2010). *Reţele neuronale. Stabilitate, Sincronizare, Întârzieri*, Seria Control Engineering, Universitaria, Craiova, ISBN 973-742-234-1.
4. D. Danciu, C. Ionete (2009). Synchronization problem for time-delay recurrent neural networks, *IFAC Proceedings Volumes*, Vol. 42, No. 14, pp 426–430.
5. D. Danciu, V. Răsvan (2007). Dynamics of Neural Networks -Some Qualitative Properties. *Proc. Int. Work Conference on Artificial Neural Networks* IWANN'2007, Lect. Notes on Comp. Sci., Springer, vol. 4507, pp. 8–15.
6. D. Danciu (2006). *Sisteme cu mai multe echilibre. Aplicaţii la reţele neurale*, Seria Control Engineering, Universitaria, Craiova, ISBN: 978-973-742-555-3.
7. D. Danciu, V. Răsvan. Stability Results for Cellular Neural Networks with Time Delays. (2005). *Proc. Int. Work Conference on Artificial Neural Networks* IWANN'2005, Lect. Notes on Comp. Sci., Springer, vol. 3512, pp. 366–373.
8. V. Răsvan, D. Danciu (2004). Neural networks - global behavior versus delay, *Scientific Bulletin of "Politehnica" University of Timişoara*, Transactions on Automatic Control and Computer Science, vol. 49(63), no. 2, pp. 11-14.
9. D. Danciu (2002). Qualitative Behaviour of the Time-Delay Hopfield-Type Neural Network with Time Varying Stimulus, *Annals of the University of Craiova*, Series: Electrical Engineering, vol. 26, No. 1, pp. 72-82.

CHAPTER 1

SYSTEMS WITH SEVERAL EQUILIBRIA AND THE CLASS OF DYNAMICAL NEURAL NETWORKS

1.1 Introduction. State of the art

Belonging to the *Artificial Intelligence* (AI) domain, the Neural Networks (NNs) are structures that possess "emergent computational capabilities" [94]. More precisely, NNs consist of interconnected simple computational devices (the artificial neuron) to which interconnections confer increased computational power – property which cannot be inferred from the properties of an individual element. There are two types of neural networks: 1) *Feedforward Networks* implement mappings from the input pattern space to the output space and do not display any dynamics, 2) *Recurrent Neural Networks* (RNNs) have, due to their cyclic interconnections and to the neurons' nonlinear activation functions, a very rich dynamical behavior including stable and unstable fixed points, limit cycles and even a chaotic behavior.

The mathematical models for RNNs arise either from modeling various biological systems or from designing artificial neural devices for solving certain tasks. In the second case, the structure of such dynamical system is induced by the "learning process" that establishes the network synaptic weights. This first stage, which gives the mathematical model for a RNN, aims at the "global pattern formation" without considering the system's qualitative properties such as stability and a "good" global behavior. Thus, this *a posteriori* induced dynamics may not have the required properties that, on the other hand, ensure a proper operation. Therefore these properties have to be checked separately after the design stage.

The most useful and investigated behaviors of the RNNs (Hopfield, CNNs, associative memories, Cohen-Grossberg, KWTA networks) are those concerning the fixed points dynamics. From the dynamical point of view, a specific feature of RNNs is that their state space consists of multiple equilibria. This characteristic grants to the neural networks their computational and problem solving capabilities. On the other hand, for systems with multiple equilibria (SME), the usual local concepts of stability (Lyapunov, asymptotic, exponential) are important but not sufficient for an adequate description. We have to take also into consideration those properties which describe the global behavior of such systems. Consequently, for RNNs with multiple nonlinearities and equilibria, the analysis has to be done within the both frameworks of *Stability theory* and *Qualitative theory of systems with several equilibria*. The results of such studies give conditions to be fulfilled by the network parameters in order the system to have the desirable dynamical properties and, these conditions have to be checked after the functional design of the neural network structure.

1.2 Systems with several equilibria. The theoretical background: notions and basic results

The *Qualitative Theory of Systems with Several Equilibria* starts from the paper of Moser [128] and has been developed in a comprehensive way by Yakubovich, Leonov and their co-workers [71, 117]. Interesting references in the field are also the papers of V.M.Popov [142, 143] and, in the context of integral and integro-differential equations, the publications of Corduneanu [33], Halanay [81], Nohel and Shea [131]. We shall recall in the sequel the basic concepts of the framework of the QTSSE, as introduced in [120].

Consider the system

$$\dot{x} = f(t,x) \tag{1.1}$$

with $f : \mathbb{R}_+ \times \mathbb{R}^n \to \mathbb{R}^n$ continuous and locally Lipschitz continuous in the second argument.

Definition 1.1. a) Any constant solution of (1.1) is called equilibrium. The set of equilibria $\mathscr{E}$ is called stationary set. b) A solution of (1.1) is called convergent if it approaches asymptotically some equilibrium: $\lim_{t\to\infty} x(t) = c \in \mathscr{E}$ c) A solution is called quasi-convergent if it approaches asymptotically the stationary set $\mathscr{E}$: $\lim_{t\to\infty} d(x(t),\mathscr{E}) = 0$, where $d(\cdot,\mathscr{E})$ denotes the distance from a point to the set $\mathscr{E}$.

Definition 1.2. The system (1.1) is called: a) monostable, if every bounded solution is convergent; b) quasi-monostable, if every bounded solution is quasi-convergent; c) gradient-like, if every solution is convergent; d) quasi-gradient-like, if every solution is quasi-convergent.

Since there are also other terms designating the above qualitative behaviors [157] in the rest of this part of the manuscript we shall use the following notions:

a) *dichotomy* – all bounded solutions tend to the equilibrium set
b) *global asymptotics* – all solutions tend to the equilibrium set
c) *gradient-like behavior* – the set of equilibria is stable in the sense of Lyapunov and any solution tends asymptotically to some equilibrium point.

The Lyapunov-like results of [120] for systems with multiple equilibria are:

Lemma 1.1. *Consider the nonlinear system*

$$\dot{x} = f(x)\ ,\ x \in \mathbb{R}^n \tag{1.2}$$

and its equilibria set $\mathscr{E} = \{c \in \mathbb{R}^n : f(c) = 0\}$. *Suppose there exists* $V : \mathbb{R}^n \mapsto \mathbb{R}$ *continuous with the following properties: i)* $V^*(t) = V(x(t))$ *is non-increasing along the solutions of (1.2); ii) if* $V(x(t)) \equiv const.$ *for some bounded on* $\mathbb{R}$ *solution of (1.2), then* $x(t) \equiv c$. *Then the system (1.2) is dichotomic.*

Lemma 1.2. *If the assumptions of Lemma 1.1 hold and either* $\lim_{|x|\to\infty} V(x) = +\infty$ *or all solutions of the system are bounded, then system (1.2) has global asymptotics i.e. each solution approaches asymptotically (for* $t \to \infty$*) the stationary set* $\mathscr{E}$.

Lemma 1.3. *If the assumptions of Lemma 1.2 hold and the set* $\mathscr{E}$ *is discrete (i.e. it consists of isolated equilibria only) then the system (1.2) is gradient-like (i.e. each solution approaches asymptotically some equilibrium that is* $\lim_{t\to\infty} x(t,x_0) = c \in \mathscr{E}$*).*

Remark 1.1. (Moser [128]) Consider the rather general nonlinear autonomous system

$$\dot{x} = -f(x)\ ,\ x \in \mathbb{R}^n \tag{1.3}$$

where $f(x) = gradG(x)$ and $G : \mathbb{R}^n \mapsto \mathbb{R}$ is such that: i)$\lim_{|x|\to\infty} G(x) = +\infty$ and ii) the number of its critical points is finite. Then any solution of (1.3) approaches asymptotically one of the equilibria (which is also a critical point of G – where its gradient, i.e. f vanishes) and thus the system's behavior is gradient-like.

Considering this framework for qualitative analysis of systems with multiple equilibria, the next chapters of Part I will study some prototype neural networks belonging to the class of dynamical neural networks with several equilibria.

1.3 Recurrent neural networks as nonlinear dynamical systems with several nonlinearities

We shall present now the mathematical models and basic features of those RNNs that will be further analyzed from the point of view of their qualitative properties.

1.3.1 Hopfield neural networks (HNN)

Introduced by J.J. Hopfield in 1982 [94], the Hopfield neural network is an one layer of fully interconnected artificial neurons. The most used mathematical model within the literature for the HNN reads as

$$\begin{aligned} \dot{x}_i &= -a_i x_i + \sum_{j=1}^{n} w_{ij} y_j + I_i\ ,\ i = \overline{1,n} \\ y_i &= f_i(x_i) \end{aligned} \tag{1.4}$$

where x_i is the neuron state, y_i is its output, I_i is the constant external stimulus or a bias, $a_i > 0$ is the rate of the passive decay of the neuron's membrane potential x_i to the resting state, $w_{ij} \in \mathbb{R}$ are the weights of the synaptic interconnections between the i^{th} neuron and the other neurons within the network where the positive values indicate excitatory connections while the negative ones show inhibitory effects for the current neuron i, n is the number of the neurons within the network.

The neuron's activation function is a sigmoid function – a nonlinear, continuous, bounded, monotonically nondecreasing and globally Lipschitzian function, i.e. it verifies

$$0 \leq \frac{f(x_1) - f(x_2)}{x_1 - x_2} \leq L \tag{1.5}$$

thus, satisfying a sector condition of the form

$$0 \leq \frac{f(x)}{x} \leq L\ ,\ f(0) = 0 \tag{1.6}$$

with $L > 0$ the Lipschitz constant.

Hopfield neural networks can be used as classifiers, optimizers as well as associative memories. The network functioning supposes the simultaneous application of the input pattern to the neurons. The neurons outputs will asynchronously and randomly activate the network neurons until a global steady state will be attained which will finally give the network output pattern $y = [y_1\ \dots\ y_n]^T$.

1.3.2 The competitive Cohen–Grossberg neural networks

The competitive neural networks, introduced by Cohen and Grossberg in 1983 [31], models the biological visual system. From the topological point of view, these networks are auto-associative, based on lateral inhibitory connections $c_{ij} < 0,\ i \neq j$ and a single excitatory connection $c_{ii} > 0$. The dynamics of the Cohen-Grossberg model is described by the equations

$$\dot{x}_i = a_i(x_i)\left[b_i(x_i) - \sum_{j=1}^{n} c_{ij} d_j(x_j)\right]\ ,\ i = \overline{1,n} \tag{1.7}$$

with $c_{ij} = c_{ji}$ (the symmetry condition).

If the functions $a_i(x_i)$ are constant and $b_i(x_i)$ are linear, then (1.7) becomes an auto-associative model with an additive activation dynamics, as the Hopfield neural network is. If, on the other hand, the functions $a_i(x_i)$ are linear and $b_i(x_i)$ are nonlinear, then the activation dynamics is multiplicative described by [31]

$$\dot{x}_i = -A_i x_i + (B_i - C_i x_i)[I_i + f_i(x_i)] - (D_i x_i + E_i)\left[J_i + \sum_{j=1\ j\neq i}^{n} F_{ij} g_j(x_j)\right]\ ,\ i = \overline{1,n} \tag{1.8}$$

where the first term describes the passive decay of the activity x_i with the rate $A_i > 0$, the second term accounts for the excitatory effect of the external stimulus I_i and of the i^{th} neuron self-feedback $y_i = f_j(x_i)$, the third term refers to the inhibitory effects of the external stimulus J_i and of the outputs-feedback $y_j = g_j(x_j)$ from the all other network neurons. It is shown in [77, 78] that in the multiplicative competitive networks each neural activity x_i is restricted to a finite interval $[-D_i^{-1}E_i,\ C_i^{-1}B_i\ ]$ for all $t \geq 0,\ i = \overline{1,n}$ and that through an appropriate design, these networks can