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Address for correspondence:
Emil PETRE
University of Craiova, Faculty of Automation, Computers and Electronics
Al.I. Cuza Street, No. 13, RO-200585, Craiova, Romania
Phone: +40-251-438198, Fax: +40-251-438198
Email: epetre@automation.ucv.ro

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Method for Fault Detection

Eugen Iancu*, Sergiu Ivanov, Eugen Bobașu*, Emil Petre***

**Department of Automation and Electronic, University of Craiova, 107 Decebal Street, RO-200440 Craiova, Romania (e-mail: Eugen.Iancu@automation.ucv.ro, http://www.ace.ucv.ro)*
***Department of Electromechanics, Environment and Industrial Informatics, University of Craiova, 107 Decebal Street, RO-200440 Craiova*

Abstract: This study shows a method for analytical fault detection that can be applied to monitor an electric motor brushless DC. The fault detection and the isolation (FDI) problem is an inherently complex one and for this reason the immediate goals is to preserve the stability of the process and, if is possible, to control the process in a slightly degraded manner. The authors propose a practical method to detect the presence of failures of sensors using a prediction solution. It was used for this purpose a mathematical model of BLDC motor and single exponential smoothing. Also is proposed a structure to detect the presence of failures.

Keywords: Brushless DC motor, fault detection and isolation, analytical redundancy, single exponential smoothing.

1. INTRODUCTION

Changes (faults) can make the system unsafe and less reliable. Productivity of the automatic system can degrade because changes can impose performance limitations on the system and may also require frequent system shut downs for its maintenance. In order to avoid production deteriorations or damage to machines and humans, variations have to be detected as quickly as possible and decisions that stop the propagation of their effects have to be made (Nguang et al., 2005).

The necessity to obtain a diagnostic with good performances, without installing a lot of redundant and dedicated expensive equipment, forces the diagnostic tools to develop the techniques available to processing all the information that are "hidden" in the technological process. In fact which the reality of industrial systems can offer to the engineer charged to implement the monitoring functions, is usually very inadequate: poor models available, lack of redundancy, insufficient number of measures, noise on the acquired data, unmodeled disturbances, etc.

The problem of reliable fault diagnosis in dynamic processes has received great attention and a wide variety of robust approaches has been proposed and developed. Analytical redundancy–based methods have been developed to diagnose faults in linear, time invariant, dynamic systems and a wide variety of model–based approaches has been proposed (Chen and Patton, 1999).

All failure detection methods exploit redundant data, which are obtained either directly, when two or more sensors are available for measurement of a process variable, or analytically, when a process variable is estimated using the mathematical process model. These redundancy relationships may then be exploited to generate residual signals. Under normal operating conditions these residuals are "*small*" in an appropriate sense and yet display distinct patterns when failures occur.

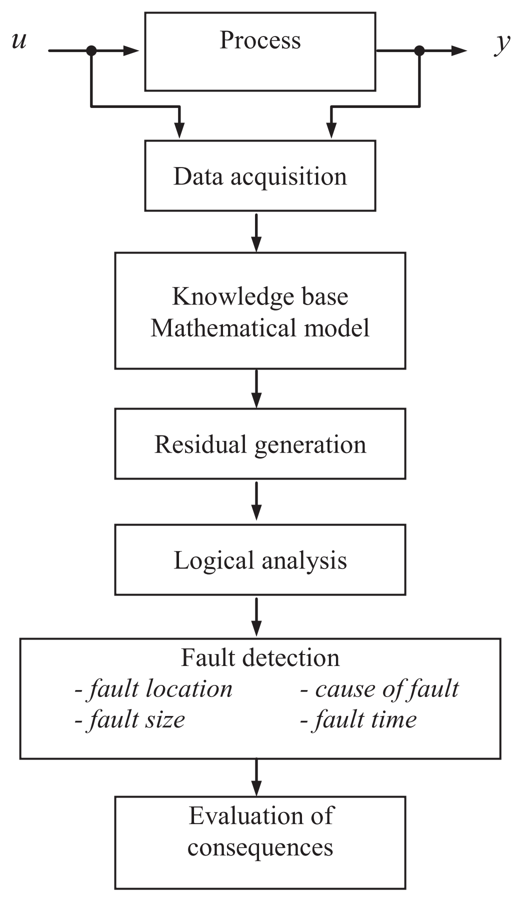

Fig. 1. The structure of the analytical diagnostic (Iancu and Vinatoru, 2005).

The failure diagnosis process consists in three stages (Fig. 1):

- Modeling of process
- Residual generation
- Residual analysis.

The residuals must be carefully examinated to determine the presence of failures (detection) and which system components have failed (isolation). In practice it is often difficult to fulfil the demands of the method for the complex diagnosed plant. Robust methods of diagnosis are therefore required, in the face of existing measurement uncertainty, disturbances and incomplete knowledge (Patton, 1994). In such cases an integrated approach using quantitative and qualitative models in diagnostic expert systems gives a good solution.

2. MATHEMATICAL MODEL OF BRUSHLESS DIRECT CURRENT MOTOR

Brushless DC motor (BLDC) is a electric motor, type synchronous, usually three-phase, having rotor with permanent magnets and stator built with concentrated or uniformly distributed windings. Specific electronic circuit is accomplished by switching semiconductor elements of the three-phase power inverter that is synchronized with the rotor position. Rotor position must be known at specific angles to align with the applied electric voltage. Usually, in order to obtain information on the rotor possition are using three Hall effect sensors, encapsulated in stator.

The mathematical model of the drive can be made using the principle ilustrated in the equivalent scheme represented in Fig.2.

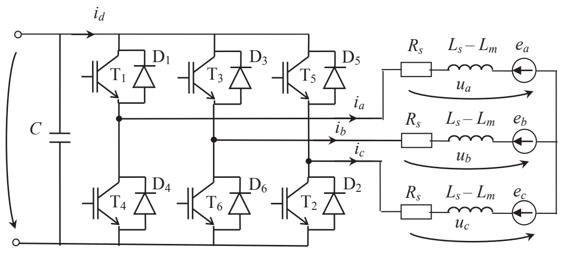

Fig. 2. Schematic diagram of the drive with brushless DC motor.

The equations of the phase voltages, are (Kennedy and Eberhart, 1995; Fedák et al., 2012):

$$u_a = R_s i_a + L_s \frac{di_a}{dt} + L_m \frac{di_b}{dt} + L_m \frac{di_c}{dt} + e_a, \tag{1}$$

$$u_b = R_s i_b + L_m \frac{di_a}{dt} + L_s \frac{di_b}{dt} + L_m \frac{di_c}{dt} + e_b, \tag{2}$$

$$u_c = R_s i_c + L_m \frac{di_a}{dt} + L_m \frac{di_b}{dt} + L_s \frac{di_c}{dt} + e_c, \tag{3}$$

where:

- u_a, u_b, u_c – instantaneous values of the phase voltages;
- e_a, e_b, e_c – the instantaneous electric phase voltages;
- i_a, i_b, i_c – the instantaneous phase currents;
- R_s, L_s – phase stator resistance and inductance, assumed the same on all phases;
- L_m – the mutual inductance.

Given the star connection of the stator windings, provided

$$i_a + i_b + i_c = 0 \tag{4}$$

allows writing equations (1) - (3) as:

$$u_a = R_s i_a + \left(L_s - L_m\right)\frac{di_a}{dt} + e_a; \tag{5}$$

$$u_b = R_s i_b + \left(L_s - L_m\right)\frac{di_b}{dt} + e_b; \tag{6}$$

$$u_c = R_s i_c + \left(L_s - L_m\right)\frac{di_c}{dt} + e_c. \tag{7}$$

Electric phase voltages depend by the position of the rotor to form a balanced three-phase system, the waveform imposed by the trapezoidal rule (Fig. 3). Thus, they can be expressed as (Fedák et al., 2012; Prasad et al., 2012):

$$e_a = \omega \cdot K_e \cdot f\left(\theta_e\right), \tag{8}$$

$$e_b = \omega \cdot K_e \cdot f\left(\theta_e - 2\pi/3\right), \tag{9}$$

$$e_c = \omega \cdot K_e \cdot f\left(\theta_e - 4\pi/3\right), \tag{10}$$

where θ_e is the electric angle of the motor, ω is the angular speed of the rotor, K_e is constant for electromotive voltages (V/(rad / sec)) and the reference function $f(\theta_e)$ for electric tension is alternatively trapezoidal with amplitude 1 (Fig. 3).

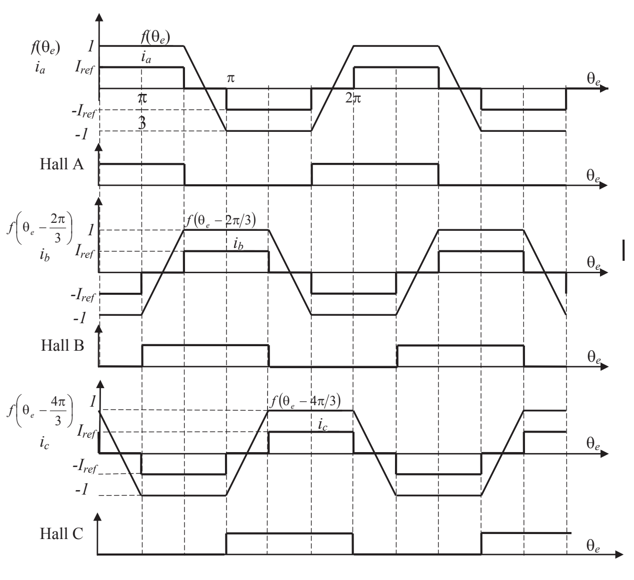

Fig. 3. Electromotive voltages, phase currents and position transducer outputs Hall

Phase alternating currents are rectangular, with alternations during the 2π/3 centered on the electric voltages (Fig. 3).

Taking the origin of the phase as shown in Fig. 3, the function $f(\theta_e)$ defined on intervals has the next expression:

$$f(\theta_e)=\begin{cases}1 & \text{for } \theta_e\in\left[0,\frac{2\pi}{3}\right]\\ 1-\frac{6}{\pi}\left(\theta_e-\frac{2\pi}{3}\right) & \text{for } \theta_e\in\left[\frac{2\pi}{3},\pi\right]\\ -1 & \text{for } \theta_e\in\left[\pi,\frac{5\pi}{3}\right]\\ -1+\frac{6}{\pi}\left(\theta_e-\frac{5\pi}{3}\right) & \text{for } \theta_e\in\left[\frac{5\pi}{3},2\pi\right]\end{cases} \tag{11}$$

To the voltage equations are added the equation of motion

$$m=J\cdot\frac{d\omega}{dt}+B\cdot\omega+m_s \tag{12}$$

where m is the electromagnetic torque developed by the motor, m_s is the static torque, J is the total moment of inertia (motor plus load), and B is the friction coefficient.

Also, the electromagnetic torque developed by the motor has the expression (Kennedy and Eberhart, 1995):

$$m=\frac{e_a i_a+e_b i_b+e_c i_c}{\omega}= \\ =K_e\left[i_a f(\theta_e-\pi/3)+i_b f(\theta_e-2\pi/3)+i_c f(\theta_e-4\pi/3)\right] \tag{13}$$

Torque is mainly influenced by the waveforms of the electromotive voltages induced in the stator of electric motors due to rotor motion. Ideally, electric tensions have trapezoidal waveforms and stator currents are rectangular (Fig. 3), resulting a constant torque. In practice, there are pulsations of torque due to design imperfections that lead to removal from electric tensions perfect form trapezoidal or due to PWM control method, switching and hysteresis. Taking into account of connection between electrical angle θ_e, mechanically angle θ_m, and the number of pole pair's p, we have the next relation:

$$\theta_e=p\cdot\theta_m, \tag{14}$$

and that

$$\omega=\frac{d\theta_m}{dt}, \tag{15}$$

Operating equations (5) - (7), (12) and (15) can be written as the state equations, respectively:

$$\frac{di_a}{dt}=-\frac{R_s}{L_s-L_m}i_a-\frac{K_e f(\theta_e)}{L_s-L_m}\omega+\frac{1}{L_s-L_m}u_a \tag{16}$$

$$\frac{di_b}{dt}=-\frac{R_s}{L_s-L_m}i_b-\frac{K_e f(\theta_e-2\pi/3)}{L_s-L_m}\omega+\frac{1}{L_s-L_m}u_b \tag{17}$$

$$\frac{di_c}{dt}=-\frac{R_s}{L_s-L_m}i_c-\frac{K_e f(\theta_e-4\pi/3)}{L_s-L_m}\omega+\frac{1}{L_s-L_m}u_c \tag{18}$$

$$\frac{d\omega}{dt}=\frac{K_e f(\theta_e)}{J}i_a+\frac{K_e f(\theta_e-2\pi/3)}{J}i_b+ \\ +\frac{K_e f(\theta_e-4\pi/3)}{J}i_c-\frac{B}{J}\omega-\frac{1}{J}m_s \tag{19}$$

$$\frac{d\theta_e}{dt}=p\omega\cdot \tag{20}$$

In the form of a matrix, the equation of state is:

$$\dot{\xi}=\mathbf{A}\cdot\xi+\mathbf{B}\cdot\mathbf{u}; \tag{21}$$

the vectors of inputs (u) and state variables (ξ) are:

$$\mathrm{u}=\begin{bmatrix}u_a & u_b & u_c & m_s\end{bmatrix} \tag{22}$$

$$\xi=\begin{bmatrix}i_a & i_b & i_c & \omega & \theta_e\end{bmatrix} \tag{23}$$

and the matrices **A** and **B** have the form:

$$\mathbf{A}=\begin{bmatrix}-\frac{R_s}{L_s-L_m} & 0 & 0 & -\frac{K_e f(\theta_e)}{L_s-L_m} & 0\\ 0 & -\frac{R_s}{L_s-L_m} & 0 & -\frac{K_e f(\theta_e-2\pi/3)}{L_s-L_m} & 0\\ 0 & 0 & -\frac{R_s}{L_s-L_m} & -\frac{K_e f(\theta_e-4\pi/3)}{L_s-L_m} & 0\\ \frac{K_e f(\theta_e)}{J} & \frac{K_e f(\theta_e-2\pi/3)}{J} & \frac{K_e f(\theta_e-4\pi/3)}{J} & -\frac{B}{J} & 0\\ 0 & 0 & 0 & p & 0\end{bmatrix} \tag{24}$$

$$\mathbf{B}=\begin{bmatrix}\frac{1}{L_s-L_m} & 0 & 0 & 0\\ 0 & \frac{1}{L_s-L_m} & 0 & 0\\ 0 & 0 & \frac{1}{L_s-L_m} & 0\\ 0 & 0 & 0 & -\frac{1}{J}\\ 0 & 0 & 0 & 0\end{bmatrix} \tag{25}$$

3. THE GENERATION OF RESIDUE DURING THE MODEL-BASED DIAGNOSIS

When the system has the actuators affected by faults, this situation can be described by the relation:

$$\begin{cases}\dot{x}(t)=\mathbf{A}x(t)+\mathbf{B}u(t)+\mathbf{B}u_d(t)\\ y(t)=\mathbf{C}x(t)+\mathbf{D}u(t)+\mathbf{D}u_d(t)\end{cases} \tag{26}$$

where $x(t)$ is the system's state, $y(t)$ is the system's output, $u(t)$ is the system's command, **A**, **B, C** and **D** are constant matrices of appropriate dimensions and the vector $u_d(t)$ represent the fault vectors for the actuators. The transfer function type input-output representation for the system is the following:

$$y(s) = H(s)u(s) + H(s)u_d(s) \tag{27}$$

where

$$H(s) = \mathbf{C}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B} + \mathbf{D} \tag{28}$$

The residue generator is a linear processor whose inputs consist in the input and output of the monitored system. This structure can be expressed mathematically so (Fig. 4):

$$r(s) = [P(s)Q(s)] \cdot \begin{bmatrix} u(s) \\ y(s) \end{bmatrix} = P(s)u(s) + Q(s)y(s) \tag{29}$$

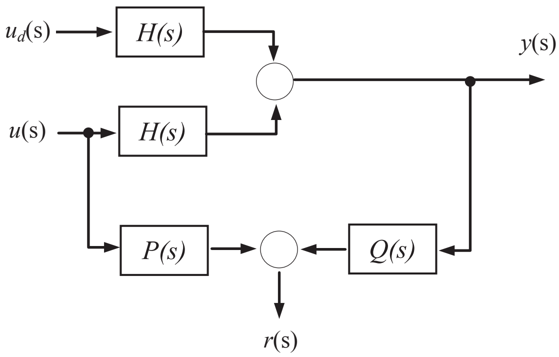

Fig. 4. The generation of the residue.

The matrixes $P(s)$ and $Q(s)$ are transfer matrixes built using linear, stabile systems. According to the definition, the residue is designed to become 0 in the case of fault absence and different from 0, in the presence of faults.

$$r(t) = 0 \text{ if and only if } u_d(t) = 0 \tag{30}$$

For the residue generator $r(s)$ to be a fault indicator, the transfer matrixes $P(s)$ and $Q(s)$ must satisfy the relation:

$$P(s) + Q(s)H(s) = 0 \tag{31}$$

When the system has the sensors affected by faults, this situation can be expressed using next structure (Fig. 5).

Let to consider a dynamical process of the form given by the linearised equations:

$$\dot{x}(t) = \mathbf{A}x(t) + \mathbf{B}u_c(t) \tag{32}$$

where $x \in \Re^n$ is the state vector, $u \in \Re^m$ is the known input vector, and **A**, **B** are constant matrices of appropriate dimensions. Similarly, the mathematical model of the process is:

$$\dot{x}_m(t) = \mathbf{A}x_m(t) + \mathbf{B}u_c(t) \tag{33}$$

The residual vector $r(t)$ is generated by the equation:

$$r(t) = x(t) - x_m(t) \tag{34}$$

$$r(t) = 0 \text{ if and only if } u_f(t) = 0 \tag{35}$$

Ideally, in absence of a fault, the residual should be zero, while when a fault is present it should be different from zero. So, a fault detection test will consist in check if the residual is zero or not.

4. EXPONENTIAL SMOOTHING METHOD

Single exponential smoothing is used for smoothing discrete time series. The efficiency of this algorithm can be attributed to its simplicity and to the capacity to adjust its responsiveness to changes in the process and its reasonable accuracy.

Let be an observed time series $X = \{x_1 \quad x_2 \quad \ldots \quad x_n\}$. Formally, the simple exponential smoothing equation takes the form (Ostertagová, 2011):

$$\tilde{x}_{i+1} = \alpha x_i + (1-\alpha)\tilde{x}_i \tag{36}$$

where x_i is the actual, known series value at moment time i, $\tilde{x}_i$ is the forecast value of the variable X at time i, $\tilde{x}_{i+1}$ is the forecast value at time i+1 and α is the smoothing constant.

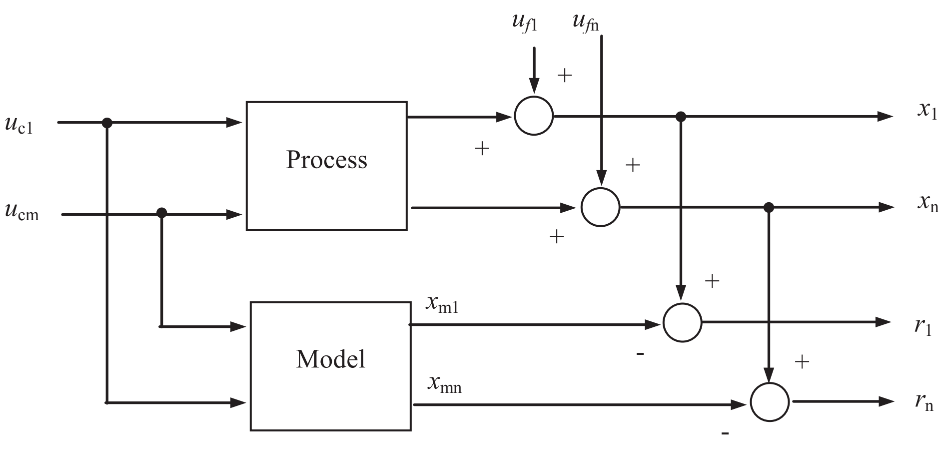

Fig. 5. Method for generates the residual vector for the sensor diagnosis.

Smoothing constant α is a selected number between zero and one, $0 < \alpha < 1$ (Brown and Meyer, 1961). When $\alpha = 1$, the original and smoothed version of the series are identical. At the other extreme, when $\alpha = 0$, the series is smoothed flat (Ostertagová, 2011). In the literature it is demonstrate the next relation (Brown and Meyer, 1961):

$$\tilde{x}_{i+1} = \alpha x_i + \alpha(1-\alpha)x_{i-1} + \alpha(1-\alpha)^2 x_{i-2} + ... \\ ... + \alpha(1-\alpha)^{i-1} x_1 = \alpha \sum_{k=0}^{i-1} (1-\alpha)^k x_{i-k} \tag{37}$$

In scientific papers are presented also *double exponential smoothing* and *triple exponential smoothing*.

From (36) we obtain:

$$\tilde{x}_{i+1} = \tilde{x}_i + \alpha(x_i - \tilde{x}_i) = \tilde{x}_i + \alpha\varepsilon_i \tag{38}$$

where ε_i represent the forecast error at time *i*. Using this error it is possible to define the following parameters (Ostertagová, 2011):

- Mean square error - *MSE*

$$MSE = \frac{1}{n} \sum_{i=1}^{n} e_i^2 \tag{39}$$

- Root mean square error - *RMSE*

$$RMSE = \sqrt{MSE} \tag{40}$$

The objective is to find an appropriate smoothing constant so that *MSE* and *RMSE* to be minimum.

5. ANALYTICAL METHOD FOR DETECTION OF FAILED SENSOR

Let to consider a dynamical process of the form given by the linearised equations (32). The method proposed by the authors is ilustred in Fig. 6. The principle is:
Step 1: The residual vector is generated.
Step 2: Using the exponential smoothing method it is calculate an anticipative value of the residual vector.
Step 3: The anticipative residual vector is used to generate an alarm (if it is different from zero) and allows the location of the faulty sensor.

6. CONCLUSIONS

The proposed method is an analytical solution, which represent a new approach for the computer-assisted diagnosis. The presented algorithm is very useful when the parameters of the process are affected by small modifications, progressive degradation, that influences the functions of the system in a slightly degraded manner.

ACKNOWLEDGMENT

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REFERENCES

Brown, R.G. and Meyer, R.F. (1961). The fundamental theory of exponential smoothing, *Operations Research*, vol. 9, pp. 673-685.

Chen, J. and Patton R.J. (1999). *Robust Model–Based Fault Diagnosis for Dynamic Systems*, Kluwer Academic Publishers.

Fedák, V., Balogh, T., and Záskalický, P. (2012). *Dynamic Simulation of Electrical Machines and Drive Systems Using MATLAB GUI - A Fundamental Tool for Scientific Computing and Engineering Applications* – Vol. 1, Vasilios Katsikis, InTech. Ed.

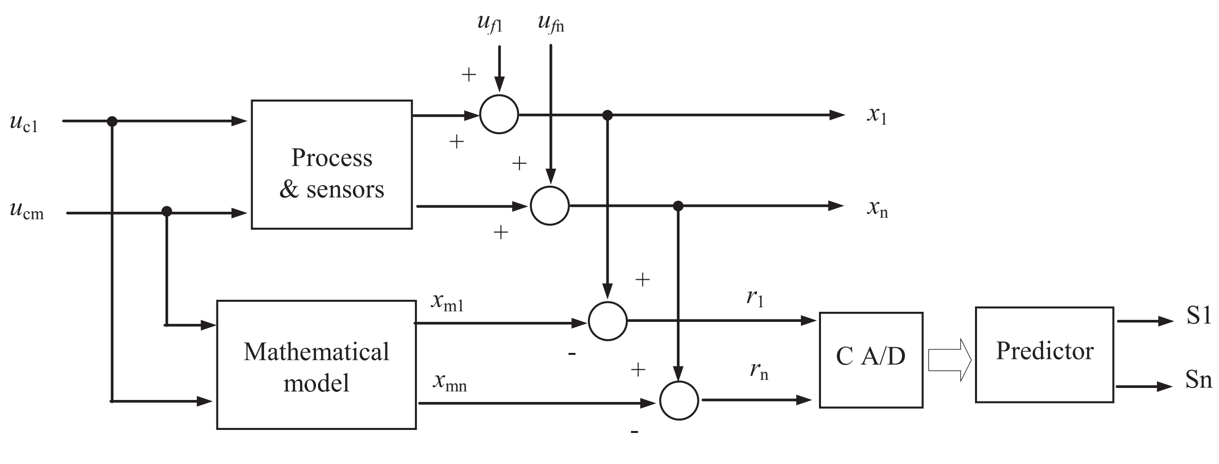

Fig. 6. Method for generates the residual vector for the sensor diagnosis.

Prasad, G., Sree Ramya, N., Prasad, P.V.N., and Tulasi Ram Das, G. (2012). Modelling and Simulation Analysis of the Brushless DC Motor by using MATLAB, *Int. J. of Innovative Technology and Exploring Engineering*, Vol. 1, Issue 5, Oct. 2012.

Iancu E. and Vinatoru M. (2005). A fault detection and isolation system using neural networks*, Proceedings of the International Conference on Control Systems and Computer Science CSCS 15*, Bucuresti, vol. 1, pp. 422-427.

Kennedy, J. and Eberhart, R.C. (1995). Particle swarm optimization. *Proc. of IEEE Int. Conference on Neural Networks*, Piscataway, NJ. pp. 1942-1948.

Nguang S.K., Shi P., Ding S. (2005). Fault detection filter for uncertain fuzzy systems: an LMI approach, *IFAC Congress*, Praha, CD proceedings.

Ostertagová, E. and Ostertag O. (2011). The simple exponential smoothing model, *Proceedings of the 4th International Conference on Modelling of Mechanical and Mechatronic Systems*, Technical University of Košice, Slovak Republic, , pp. 380-384.

Patton R.J. and Chen J. (1994). A review of parity space approaches to fault diagnosis applicable to aerospace systems, *Journal of Guidance, Control and Dynamics*, vol. 17, no. 2, pp. 278-285.

Fault Detection in Educational Kit Festo

Camelia Maican*, Gabriela Cănureci**

** Automation and Electronics Department, University of Craiova, Romania (e-mail: camelia@automation.ucv.ro).*
*** Research and Development, Engineering and Manufacturing for Automation Equipment and Systems SC IPA SA CIFATT Craiova, Romania (e-mail: gabriela.canureci@ipacv.ro).*

Abstract: In this paper is study the faults detection and localization, in educational kit Festo, using residual methods. The level control system and the faults detection structure were developed under Matlab Simulink. This faults detection structure allows us to detect two faults that can occur in the plant, separately and simultaneously. The proposed method was theoretically developed and experimentally verified on the plant model.

Keywords: control; fault detection and localization; level; residue.

1. INTRODUCTION

The plant Festo contains individual modules which can be combined in different ways and allows the level, temperature and flow control. Based on equivalence relations of flowing phenomena, one can determine the equivalence of the level controlling system within Festo plant with the level controlling system of the drums in the steam boilers within the thermal power groups (Canureci et al. 2012).

Within both plants, there can be malfunctions like:

- blocking the adjustment flap of the boiler feed water flow, or a changing transfer factor, which can be simulated on the Festo plant by modifying the flowing section of the tap on the circulation pump discharge.
- pipe breaking or stopping of a power pump on the water flow of the boiler, which lowers the boiler feed water flow according to the value generated by the controller, fact that can be simulated on the plant by activating the tap R_3, which partially controls the flow Fp directly to the second tank.

This way, the structures of detection and identification of the malfunctions that may appear on the given plant can be applied analogically to the steam boiler level control system (Iancu et al. 2003 and Vinatoru 2001).

2. THE MATHEMATICAL MODELS

The hydraulic system diagram of the educational kit FESTO is shown in Fig.1 (Canureci et al. 2012). The plant consists of two parallelepipedic transparent plastic water tanks assembled on an aluminium platform with supporting holders. The tanks are placed one in upper position the other in lower position. A water pump (P) ran by a driving motor (DM) ensures water flow from the lower tank to the upper tank through a system of pipelines, bends and connecting pipes.

At the exit of the pump (P), on the pipe there is a vertically assembled hydraulic diode of connection which prevents water leaks from the upper tank to the lower tank when the pump discharge pressure gets below the hydrostatic pressure which corresponds to the liquid level of the upper tank. A pipe system combined with two taps R_1 and R_2 allow conducting the water discharged by the pump either towards the upper tank or to the lower tank. Water flow from the upper to the lower tank is done naturally through a pipe system on which there is a tap R_3 which allows changes in the flowing section or obstructions of the pipe (Vinatoru et al. 2008).

On the upper tank there is mounted a level transducer with ultrasounds which determines an electric signal at the exit 4-20 mA DC for a fluctuation of the liquid level within the field 0-500 mm (Vinatoru et al. 2007). The liquid level control is achieved by changing the water pump flow (F_3=F_p), using the speed command of the driving motor (DM).

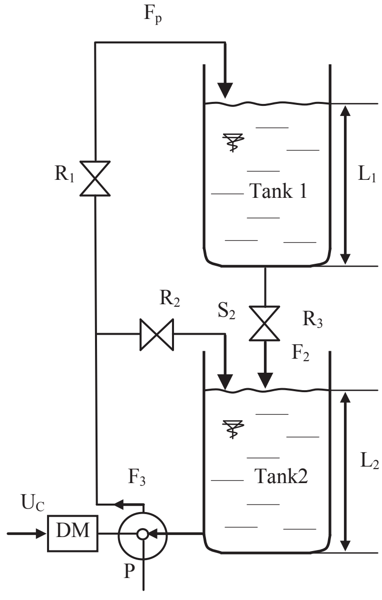

Fig. 1 The hydraulic system diagram

This is achieved by varying the supply voltage of the pump using the pump regulator, which is driven with a voltage signal in the range of 2-10 V DC.

The state variables are:

$x_1 = L_1$ the level of the tank1

$x_2 = L_2$ the level of the tank2

The input variables are:

$F_3 = F_p = U = k_p U_c$ the command to adjust the level in the tank 1

$F_2 = C_1 S_2 \sqrt{\rho g L_1} = C S_2 \sqrt{L_1}$ the evacuation flow from the tank 2.

The mathematical model of the system is determined using the mass balance equations (Vinatoru et al. 2008):

$$\frac{dL_1}{dt} = \frac{F_p(U_c) - F_2(S_2, L_1)}{A_1} = \frac{k_p U_C - C\ S_2\sqrt{L_1}}{A_1} \quad (1)$$

$$\frac{dL_2}{dt} = \frac{F_2(S_2, L_1) - F_3(U_C)}{A_2} = \frac{C\ S_2\sqrt{L_1} - k_p U_C}{A_2} \quad (2)$$

where C is a coefficient depending on the viscosity of the fluid loss and sectional shape of the flowing, S_2 sectional area of the valve transitions between the two tanks, A_1 and A_2 cross-sectional area of the Tank1, respectively Tank2.

In canonical form the mathematical model is:

$$\frac{dL_1}{dt} = -\frac{C}{A_1} S_2\sqrt{L_1} + \frac{K_P}{A_1} U_C \quad (3)$$

$$\frac{dL_2}{dt} = \frac{C}{A_2} S_2\sqrt{L_1} - \frac{K_P}{A_2} U_2 \quad (4)$$

In steady state:

$$k_p U_{P0} = C\ S_{20}\sqrt{L_{10}} = F_{20} \quad (5)$$

By linearizing of the equations (3) and (4) around the steady state values, resulting linear form of the mathematical model (6) and (7), which contain the section variation ΔS_2 which can occur only under fault conditions. Under normal conditions $\Delta S_2 = 0$.

$$\Delta\dot{x}_1 = -\frac{C S_{20}}{2A_1\sqrt{L_{10}}}\Delta x_1 + \frac{k_p}{A_1}\Delta U_C - \frac{C\sqrt{L_{10}}}{A_1}\Delta S_2 \quad (6)$$

$$\Delta\dot{x}_2 = \frac{C S_{20}}{2A_1\sqrt{L_{10}}}\Delta x_1 - \frac{k_p}{A_1}\Delta U_C + \frac{C\sqrt{L_{10}}}{A_1}\Delta S_2 \quad (7)$$

where: $\Delta x_1 = \Delta L_1 = L_1 - L_{10}$

$\Delta x_2 = \Delta L_2 = L_2 - L_{20}$,

$x_{10} = L_{10}$ and $x_{20} = L_{20}$

Applying Laplace transform in zero initial conditions result:

$$\Delta x_1(s) = \frac{1}{Ts+1}\left[\frac{2x_{10}k_p}{F_{20}}\Delta U_C(s) - \frac{2x_{10}}{S_{20}}\Delta S_2(s)\right] \quad (8)$$

$$\Delta x_2(s) = \frac{A_1/A_2}{Ts+1}\left[-\frac{2x_{10}k_p}{F_{20}}\Delta U_C(s) + \frac{2x_{10}}{S_{20}}\Delta S_2(s)\right] \quad (9)$$

where: $T = \dfrac{2A_1 x_{10}}{F_{20}}$

and $A_1 = A_2 = 1{,}75\cdot 1{,}9 = 3{,}325\text{dm}^2$

$L_{10} = L_{20} = 2\text{dm}$

$F_{10} = F_{20} = 4{,}7\text{dm}^3$

3. THE FAULT DETECTION AND LOCALIZATION

We will analyze the following faults which may occur in the FESTO system:

- The pump is not running completely the command received or an additional pressure loss occurs on the above valve, which is mounted after the pump, and this makes the pump flow to be influenced by fault, therefore be no longer proportional to the control voltage:

$$\Delta F_p = k_p\left(\Delta U_{Cn} + \Delta U_d\right) \quad (10)$$

where ΔU_{Cn} is pump control signal and ΔU_d the equivalent in the control signal of the actuator's fault.

- Modification of the flow section S_2 due either to variation of the flow regime, by plugging the crossing section or additional breakings of pipeline, which is equivalent to an increase of S_2; this makes as the exhaust flow from the Tank1 in Tank 2 to be of the form:

$$\Delta F_2 = C\sqrt{L_{10}}\left(\Delta S_{2n} + \Delta S_{2d}\right) \quad (11)$$

Where ΔS_{2n} is the value of the section under normal functioning and ΔS_{2d} is equivalent value of the section, caused by the fault.

In these circumstances, by introducing the faults specified in equations (6) and (7), replacing

$$\Delta U_C = \Delta U_{Cn} + \Delta U_d$$

$$\Delta S_2 = \Delta S_{2n} + \Delta S_{2d}$$

we obtain:

$$\Delta\dot{x}_1 = -\frac{C S_{20}}{2A_1\sqrt{L_{10}}}\Delta x_1 + \frac{k_p}{A_1}\left(\Delta U_C + \Delta U_d\right) - \frac{C\sqrt{L_{10}}}{A_1}\left(\Delta S_{2n} + \Delta S_{2d}\right) \quad (12)$$