ANNALS OF THE UNIVERSITY OF CRAIOVA
ANNALES DE L'UNIVERSITÉ DE CRAIOVA

ANALELE UNIVERSITĂȚII DIN CRAIOVA

SERIA: INGINERIE ELECTRICĂ
SERIE: ELECTRICAL ENGINEERING
SÉRIE: INGÉNIERIE ÉLECTRIQUE
Anul/Year/Année 42
No. 42, Vol. 42, Issue 2, 2018

December 2018

ISSN 1842 - 4805

EDITURA UNIVERSITARIA

ANNALS OF THE UNIVERSITY OF CRAIOVA

13, A.I. Cuza Str., CRAIOVA 200585
ROMANIA

We exchange publications with similar institutions of country and from abroad

ANNALES DE L'UNIVERSITÉ DE CRAIOVA

Rue A.I. Cuza, No. 13, CRAIOVA 200585
ROUMANIE

On fait des échanges des publications avec les institutions similaires du pays et de l'étranger

This journal is published by the Faculty of Electrical Engineering from the University of Craiova. The authors are fully responsible for the originality of their papers and for accuracy of their notes.

This volume has benefited by the financial support of the Association for Support of Engineering Education

REVIEWERS COMMITTEE

CPC–Based Reactive Compensation of Linear Loads Supplied with Asymmetrical Nonsinusoidal Voltage

Leszek S. Czarnecki, Life Fellow, IEEE, Prashanna D. Bhattarai, Member, IEEE
School of Electrical Engineering and Computer Science, Louisiana State University,
Baton Rouge, USA, lsczar@cox.net, www.lsczar.info

Abstract – **The paper presents a method of calculation of LC parameters of reactive balancing compensators of linear time-invariant (LTI) loads supplied with a nonsinusoidal and asymmetrical voltage. The method is based on the Currents' Physical Components (CPC) – based power theory. Complete compensation of the reactive and unbalanced currents in the presence of the supply voltage harmonics can require compensators of very high complexity. The paper presents a method of calculation of LC parameters of reactive balancing compensators with a reduced complexity. The method is illustrated with a numerical example.**

Keywords: *Compensation, power theory, power definitions, unbalanced loads, Currents' Physical Components, CPC.*

I. INTRODUCTION

A great majority of three-phase loads are balanced and are supplied by a voltage with negligible asymmetry and waveform distortion. Compensation of only the reactive power might be needed for the power factor improvement of such loads.

Distribution systems in manufacturing plants supply not only balanced three-phase loads but also aggregates of single-phase loads or traction systems, which usually take energy from only a single line of a three-phase system, however. Also, ac arc furnaces in metallurgic plants could be sources of asymmetry and distortion.

Switching compensators and reactive compensators can be used for the reactive power compensation and the load balancing. Switching compensators, due to their natural adaptability are advantageous over reactive compensators. Their power is confined by switching power of transistors needed for their construction, however. Reactive compensators do not have such limitations. They can be built as adaptive devices, using switches for changing compensator parameters. When thyristors are used for providing adaptability [3], then the compensator power is confined by thyristor's switching power. This power is at least one order higher than such a power of transistors.

The LC parameters of a compensator can be calculated in optimization procedures [13], Such procedures provide us with the compensator parameters, but do not contribute to the general knowledge on a compensator design, however. Also, a lot of calculation could be required by such optimization procedures. Therefore, an optimiza-tion-based approach could be disadvantageous when the compensator should have an adaptive property.

Calculation of the LC parameters of a compensator from algebraic formulae could be much faster than their calculation in an optimization procedure. Knowledge of the power properties of the load at nonsinusoidal and asymmetrical voltage is needed for developing of such algebraic formulae, however.

Because most of the electric energy is transferred and used in three-phase systems, the development of methods of reactive compensators design focused substantial attention in electrical engineering community [4-6, 9, 12 - 17]. The first reactive balancing compensator was developed by Steinmetz in 1917 [1], known [11] as a "Steinmetz' circuit". It was a compensator developed for operation at symmetrical and sinusoidal voltage.

A method of calculation of LC parameters of reactive balancing compensators for three-phase circuits with nonsinusoidal supply voltage was developed in a frame of the Currents' Physical Components (CPC) - based power theory (PT) in [8]. This method was developed with the assumption that the supply voltage was symmetrical, however. This paper presents a method of calculation of LC parameters of reactive balancing compensators without this assumption, meaning at asymmetrical voltage.

The method presented in this paper is based on the CPC – based PT, [18, 19], as drafted below.

II. CPC OF THREE-PHASE LTI LOADS AT ASYMMETRICAL NONSINUSOIDAL VOLTAGE

Let a linear time-invariant (LTI) load is supplied by a three-wire line, as shown in Fig. 1, with nonsinusoidal and asymmetrical and voltage.

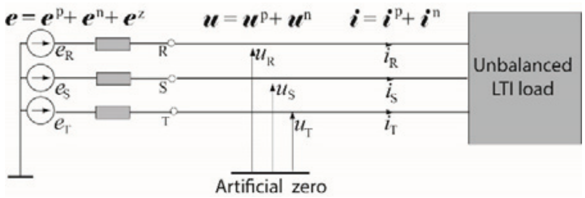

Fig. 1. LTI load supplied by a three-wire line.

Symbols $\boldsymbol{u}$ and $\boldsymbol{i}$ in Fig. 1 denote the voltage and current vectors

$$\boldsymbol{u}(t) \overset{\text{df}}{=} [u_R(t),\ u_S(t),\ u_T(t)]^T, \qquad \boldsymbol{i}(t) \overset{\text{df}}{=} [i_R(t),\ i_S(t),\ i_T(t)]^T.$$

When the load voltage contains harmonics of the order n from a set N then it can be presented in the form

$$\boldsymbol{u}(t)=\sum_{n\in N}\boldsymbol{u}_n(t)=\sqrt{2}\mathrm{Re}\sum_{n\in N}\begin{bmatrix}\boldsymbol{U}_{\mathrm{R}n}\\ \boldsymbol{U}_{\mathrm{S}n}\\ \boldsymbol{U}_{\mathrm{T}n}\end{bmatrix}e^{jn\omega_1 t}=\sqrt{2}\mathrm{Re}\sum_{n\in N}\boldsymbol{U}_n e^{jn\omega_1 t}\,. \quad (1)$$

Similarly, the load current

$$\boldsymbol{i}(t)=\sum_{n\in N}\boldsymbol{i}_n(t)=\sqrt{2}\,\mathrm{Re}\sum_{n\in N}\begin{bmatrix}\boldsymbol{I}_{\mathrm{R}n}\\ \boldsymbol{I}_{\mathrm{S}n}\\ \boldsymbol{I}_{\mathrm{T}n}\end{bmatrix}e^{jn\omega_1 t}=\sqrt{2}\,\mathrm{Re}\sum_{n\in N}\boldsymbol{I}_n e^{jn\omega_1 t}\,. \quad (2)$$

The load current can be decomposed into four Currents' Physical Components (CPC)

$$\boldsymbol{i}=\boldsymbol{i}_{\mathrm{a}}+\boldsymbol{i}_{\mathrm{r}}+\boldsymbol{i}_{\mathrm{s}}+\boldsymbol{i}_{\mathrm{u}} \quad (3)$$

defined as follows

$$\boldsymbol{i}_{\mathrm{a}}\stackrel{\mathrm{df}}{=}G_{\mathrm{b}}\,\boldsymbol{u}=\sqrt{2}\,\mathrm{Re}\sum_{n\in N}G_{\mathrm{b}}\boldsymbol{U}_n e^{jn\omega_1 t} \quad (4)$$

is the ***active current***. In this formula $G_{\mathrm{b}}=P/||\boldsymbol{u}||^2$ is the conductance of a balanced resistive load which is equivalent with respect to the active power P to the original load. Symbol $||\boldsymbol{u}||$ denotes a three-phase rms value of the load voltage, defined in [8] as,

$$||\boldsymbol{u}||=\sqrt{||u_{\mathrm{R}}||^2+||u_{\mathrm{S}}||^2+||u_{\mathrm{T}}||^2}\,. \quad (5)$$

The next component

$$\boldsymbol{i}_{\mathrm{r}}\stackrel{\mathrm{df}}{=}\sum_{n\in N}\boldsymbol{i}_{\mathrm{r}n}=\sqrt{2}\,\mathrm{Re}\sum_{n\in N}jB_{\mathrm{b}n}\boldsymbol{U}_n e^{jn\omega_1 t} \quad (6)$$

is the ***reactive current***, where $B_{\mathrm{b}n}=-Q_n/||\boldsymbol{u}_n||^2$ is the ***equivalent susceptance*** of a balanced load which is equivalent to the original load with respect to the harmonic reactive power Q_n. The third component

$$\boldsymbol{i}_{\mathrm{s}}\stackrel{\mathrm{df}}{=}\sqrt{2}\mathrm{Re}\sum_{n\in N}(G_{\mathrm{b}n}-G_{\mathrm{b}})\boldsymbol{U}_n e^{jn\omega_1 t} \quad (7)$$

is the ***scattered current***, where $G_{\mathrm{b}n}=P_n/||\boldsymbol{u}_n||^2$ is the ***equivalent conductance*** of a balanced load which is equivalent to the original load with respect to the active power for n^{th} order harmonic P_n. The last component

$$\boldsymbol{i}_{\mathrm{u}}\stackrel{\mathrm{df}}{=}\boldsymbol{i}-(\boldsymbol{i}_{\mathrm{a}}+\boldsymbol{i}_{\mathrm{r}}+\boldsymbol{i}_{\mathrm{s}}) \quad (8)$$

is the ***unbalanced current***.

All Currents' Physical Components are mutually orthogonal, so that the three-phase rms value of the load current, which can be calculated having the rms values $||i_{\mathrm{R}}||$, $||i_{\mathrm{S}}||$ and $||i_{\mathrm{T}}||$, namely

$$||\boldsymbol{i}||=\sqrt{||i_{\mathrm{R}}||^2+||i_{\mathrm{S}}||^2+||i_{\mathrm{T}}||^2} \quad (9)$$

can also be calculated having three-phase rms values of CPC, namely

$$||\boldsymbol{i}||=\sqrt{||\boldsymbol{i}_{\mathrm{a}}||^2+||\boldsymbol{i}_{\mathrm{r}}||^2+||\boldsymbol{i}_{\mathrm{s}}||^2+||\boldsymbol{i}_{\mathrm{u}}||^2} \quad (10)$$

where

$$||\boldsymbol{i}_{\mathrm{a}}||=G_{\mathrm{b}}||\boldsymbol{u}|| \quad (11)$$

$$||\boldsymbol{i}_{\mathrm{r}}||=\sqrt{\sum_{n\in N}B_{\mathrm{b}n}^2\,||\boldsymbol{u}_n||^2} \quad (12)$$

$$||\boldsymbol{i}_{\mathrm{s}}||=\sqrt{\sum_{n\in N}(G_{\mathrm{b}n}-G_{\mathrm{b}})^2\,||\boldsymbol{u}_n||^2} \quad (13)$$

$$||\boldsymbol{i}_{\mathrm{u}}||=\sqrt{||\boldsymbol{i}||^2-(||\boldsymbol{i}_{\mathrm{a}}||^2+||\boldsymbol{i}_{\mathrm{r}}||^2+||\boldsymbol{i}_{\mathrm{s}}||^2)}\,. \quad (14)$$

The last formula enables calculation the unbalanced current three-phase rms value only indirectly. It is not expressed in terms of the load parameters. This could be satisfactory when only this value is a matter of interest, but not for compensation purposes. For the design of a balancing compensator that could reduce this current, its dependence on the circuit parameters is needed.

III. UNBALANCED ADMITTANCE

When the load voltage $\boldsymbol{u}$ is asymmetrical then, as it results from (4), (6) and (7), the active, reactive and scattered currents replicate the load voltage asymmetry. The energy provider, not the customer is responsible for this asymmetry of the load current. When the load is purely resistive and balanced, it is a unity power factor load, independently on the load current asymmetry.

When the load is unbalanced then the load current $\boldsymbol{i}$ does not replicate the voltage asymmetry, however. The difference between the load current $\boldsymbol{i}$, and currents that replicate the load voltage asymmetry, meaning the active, reactive and scattered currents $\boldsymbol{i}_{\mathrm{a}}$, $\boldsymbol{i}_{\mathrm{r}}$ and $\boldsymbol{i}_{\mathrm{s}}$, is the unbalanced current $\boldsymbol{i}_{\mathrm{u}}$, as defined by formula (8).

Since the CPC are sums of harmonics, the n^{th} order harmonic of the unbalanced current is equal to

$$\boldsymbol{i}_{\mathrm{u}n}=\boldsymbol{i}_n-(\boldsymbol{i}_{\mathrm{a}n}+\boldsymbol{i}_{\mathrm{r}n}+\boldsymbol{i}_{\mathrm{s}n})\,. \quad (15)$$

This requires that vectors of the n^{th} order harmonic complex rms (crms) values of line currents satisfy the equation

$$\begin{aligned}\boldsymbol{I}_{\mathrm{u}n}&=\boldsymbol{I}_n-[G_{\mathrm{b}}\boldsymbol{U}_n+jB_{\mathrm{b}n}\boldsymbol{U}_n+(G_{\mathrm{b}n}-G_{\mathrm{b}})\boldsymbol{U}_n]=\\&=\boldsymbol{I}_n-Y_{\mathrm{b}n}\boldsymbol{U}_n\end{aligned} \quad (16)$$

where

$$Y_{\mathrm{b}n}\stackrel{\mathrm{df}}{=}G_{\mathrm{b}n}+jB_{\mathrm{b}n}=\frac{P_n-jQ_n}{||\boldsymbol{u}_n||^2}\,. \quad (17)$$

With respect to harmonics of the line currents $i_{\mathrm{R}n}$, $i_{\mathrm{S}n}$, and $i_{\mathrm{T}n}$, the original load is equivalent to a load in Δ structure, with line-to-line admittances $\boldsymbol{Y}_{\mathrm{RS}n}$, $\boldsymbol{Y}_{\mathrm{ST}n}$ and $\boldsymbol{Y}_{\mathrm{TR}n}$. In fact [10], there is an infinite number of such equivalent loads, so that one of line-to-line admittances can have any value, in particular, zero, and remaining ones can be calculated accordingly.

The order n of a harmonic is in the following analysis irrelevant for results so that for reducing symbols complexity, the index "n" is below neglected. It will be restored when this order would affect the results obtained.

The vector of crms values of the unbalanced current for a harmonic of any order n is

$$\begin{aligned}\boldsymbol{I}_{\mathrm{u}}=\boldsymbol{I}-Y_{\mathrm{b}}\boldsymbol{U}&=\begin{bmatrix}\boldsymbol{Y}_{\mathrm{RS}}(\boldsymbol{U}_{\mathrm{R}}-\boldsymbol{U}_{\mathrm{S}})-\boldsymbol{Y}_{\mathrm{TR}}(\boldsymbol{U}_{\mathrm{T}}-\boldsymbol{U}_{\mathrm{R}})\\ \boldsymbol{Y}_{\mathrm{ST}}(\boldsymbol{U}_{\mathrm{S}}-\boldsymbol{U}_{\mathrm{T}})-\boldsymbol{Y}_{\mathrm{RS}}(\boldsymbol{U}_{\mathrm{R}}-\boldsymbol{U}_{\mathrm{S}})\\ \boldsymbol{Y}_{\mathrm{TR}}(\boldsymbol{U}_{\mathrm{T}}-\boldsymbol{U}_{\mathrm{R}})-\boldsymbol{Y}_{\mathrm{ST}}(\boldsymbol{U}_{\mathrm{S}}-\boldsymbol{U}_{\mathrm{T}})\end{bmatrix}-Y_{\mathrm{b}}\boldsymbol{U}=\\&=(Y_{\mathrm{e}}-Y_{\mathrm{b}})\boldsymbol{U}+\boldsymbol{Y}\,\boldsymbol{U}\end{aligned} \quad (18)$$

where

$$Y_{\mathrm{e}}\stackrel{\mathrm{df}}{=}\boldsymbol{Y}_{\mathrm{ST}}+\boldsymbol{Y}_{\mathrm{TR}}+\boldsymbol{Y}_{\mathrm{RS}} \quad (19)$$

is the ***equivalent admittance*** of the load for any harmonic order, and

$$\boldsymbol{Y} \stackrel{\text{df}}{=} -\begin{bmatrix} \boldsymbol{Y}_{\text{ST}}, & \boldsymbol{Y}_{\text{RS}}, & \boldsymbol{Y}_{\text{TR}} \\ \boldsymbol{Y}_{\text{RS}}, & \boldsymbol{Y}_{\text{TR}}, & \boldsymbol{Y}_{\text{ST}} \\ \boldsymbol{Y}_{\text{TR}}, & \boldsymbol{Y}_{\text{ST}}, & \boldsymbol{Y}_{\text{RS}} \end{bmatrix}. \tag{20}$$

A three-phase vector of the load voltages can be decomposed into vectors of voltages of the positive sequence $\boldsymbol{u}^{\text{p}}$ and negative sequence $\boldsymbol{u}^{\text{n}}$. It applies to each voltage harmonic, so that

$$\boldsymbol{u} = \boldsymbol{u}^{\text{p}} + \boldsymbol{u}^{\text{n}} = \sqrt{2}\,\text{Re}\sum_{n\in N}(\boldsymbol{U}_n^{\text{p}} + \boldsymbol{U}_n^{\text{n}})e^{jn\omega_1 t} \tag{21}$$

where, independently on the harmonic order n

$$\boldsymbol{U}^{\text{p}} = \mathbf{1}^{\text{p}}U^{\text{p}}, \qquad \boldsymbol{U}^{\text{n}} = \mathbf{1}^{\text{n}}U^{\text{n}} \tag{22}$$

with

$$\mathbf{1}^{\text{p}} \stackrel{\text{df}}{=} [1,\ \alpha^*,\ \alpha]^{\text{T}},\quad \mathbf{1}^{\text{n}} \stackrel{\text{df}}{=} [1,\ \alpha,\ \alpha^*]^{\text{T}},\quad \alpha = 1e^{j\frac{2\pi}{3}} \tag{23}$$

while $\boldsymbol{U}^{\text{p}}$ and $\boldsymbol{U}^{\text{n}}$ are the crms values of the load voltage symmetrical components of the positive and negative sequence, equal to

$$\begin{bmatrix} \boldsymbol{U}^{\text{p}} \\ \boldsymbol{U}^{\text{n}} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1, & \alpha, & \alpha^* \\ 1, & \alpha^*, & \alpha \end{bmatrix}\boldsymbol{U}\ . \tag{24}$$

With this decomposition of the load voltage into symmetrical components, the term $\boldsymbol{Y}\boldsymbol{U}$ in (18) can be rearranged to

$$\boldsymbol{Y}\boldsymbol{U} = -\boldsymbol{Y}\,[\mathbf{1}^{\text{p}}\boldsymbol{U}^{\text{p}} + \mathbf{1}^{\text{n}}\boldsymbol{U}^{\text{n}}] = \mathbf{1}^{\text{n}}\boldsymbol{Y}^{\text{p}}\boldsymbol{U}^{\text{p}} + \mathbf{1}^{\text{p}}\boldsymbol{Y}^{\text{n}}\boldsymbol{U}^{\text{n}} \tag{25}$$

where

$$\boldsymbol{Y}_{\text{u}}^{\text{p}} \stackrel{\text{df}}{=} -(\boldsymbol{Y}_{\text{ST}} + \alpha\boldsymbol{Y}_{\text{TR}} + \alpha^*\boldsymbol{Y}_{\text{RS}}) \tag{26}$$

$$\boldsymbol{Y}_{\text{u}}^{\text{n}} \stackrel{\text{df}}{=} -(\boldsymbol{Y}_{\text{ST}} + \alpha^*\boldsymbol{Y}_{\text{TR}} + \alpha\boldsymbol{Y}_{\text{ST}}) \tag{27}$$

are ***unbalanced admittances*** of the load for the voltage of the positive and negative sequence, respectively.

Eventually, formulae (18) and (25) result in the vector of crms values of unbalanced current harmonics. To emphasize that it applies to individual harmonics, the index of the harmonic order "n" is now restored, i.e.,

$$\boldsymbol{I}_{\text{u}n} = \boldsymbol{Y}_{\text{d}n}\boldsymbol{U}_n + \mathbf{1}^{\text{n}}\boldsymbol{Y}_{\text{u}n}^{\text{p}}\boldsymbol{U}_n^{\text{p}} + \mathbf{1}^{\text{p}}\boldsymbol{Y}_{\text{u}n}^{\text{n}}\boldsymbol{U}_n^{\text{n}} \tag{28}$$

where

$$\boldsymbol{Y}_{\text{d}n} = \boldsymbol{Y}_{\text{e}n} - \boldsymbol{Y}_{\text{b}n}\ . \tag{29}$$

There is no difference between $\boldsymbol{Y}_{\text{e}n}$ and $\boldsymbol{Y}_{\text{b}n}$ values, i.e., $\boldsymbol{Y}_{\text{d}n} = 0$, at symmetrical load voltages, but not at the voltage asymmetry. Therefore, admittance $\boldsymbol{Y}_{\text{d}n}$, is referred to as the ***voltage asymmetry dependent admittance.***

The equivalent admittance $\boldsymbol{Y}_{\text{e}n}$ and unbalanced admittances $\boldsymbol{Y}_n^{\text{p}}$ and $\boldsymbol{Y}_n^{\text{n}}$ can be expressed with (19), (26) and (27) directly in terms of the line-to-line admittances $\boldsymbol{Y}_{\text{RS}n}$, $\boldsymbol{Y}_{\text{ST}n}$ and $\boldsymbol{Y}_{\text{TR}n}$ of the load equivalent circuit. The unbalanced admittance $\boldsymbol{Y}_{\text{b}n}$ and consequently, also admittance $\boldsymbol{Y}_{\text{d}n}$, are not expressed explicitly in terms of these admittances, however. To express the load unbalanced current in terms of the load admittances, we have to express first the balanced admittance $\boldsymbol{Y}_{\text{b}n}$ for the n^{th} order harmonic in terms of these parameters. This admittance, defined by (17), with index "n" neglected for simplicity, can be rearranged as follows

$$\boldsymbol{Y}_{\text{b}} = \frac{P - jQ}{||\boldsymbol{u}||^2} = \frac{\boldsymbol{C}^*}{||\boldsymbol{u}||^2} = \frac{\boldsymbol{C}_{\text{RS}}^* + \boldsymbol{C}_{\text{ST}}^* + \boldsymbol{C}_{\text{TR}}^*}{||\boldsymbol{u}||^2} =$$
$$= 2\boldsymbol{Y}_{\text{e}} - \frac{3}{||\boldsymbol{u}||^2}(\boldsymbol{Y}_{\text{ST}}U_{\text{R}}^2 + \boldsymbol{Y}_{\text{TR}}U_{\text{S}}^2 + \boldsymbol{Y}_{\text{RS}}U_{\text{T}}^2) = \boldsymbol{Y}_{\text{e}} - \boldsymbol{Y}_{\text{d}}. \tag{30}$$

The symbol „$\boldsymbol{C}$" instead of a common symbol „$\boldsymbol{S}$" is used in formula (30) to avoid confusion of the magnitude of the complex power $P+jQ$ with the apparent power S, which can contain also components other than only the active and reactive powers. The equation (30) can be rearranged to the form

$$\boldsymbol{Y}_{\text{d}} = \frac{3}{||\boldsymbol{u}||^2}(\boldsymbol{Y}_{\text{ST}}U_{\text{R}}^2 + \boldsymbol{Y}_{\text{TR}}U_{\text{S}}^2 + \boldsymbol{Y}_{\text{RS}}U_{\text{T}}^2) - \boldsymbol{Y}_{\text{e}} \tag{31}$$

where

$$||\boldsymbol{u}||^2 = U_{\text{R}}^2 + U_{\text{S}}^2 + U_{\text{T}}^2 = 3(U^{\text{p}2} + U^{\text{n}2})\ . \tag{32}$$

Observe that at the voltage symmetry, $U_{\text{R}} = U_{\text{S}} = U_{\text{T}}$, and consequently, $\boldsymbol{Y}_{\text{d}} = 0$. Let us calculate the square of the load voltages rms value

$$U_{\text{R}}^2 = \boldsymbol{U}_{\text{R}}\boldsymbol{U}_{\text{R}}^* = (\boldsymbol{U}^{\text{p}} + \boldsymbol{U}^{\text{n}})(\boldsymbol{U}^{\text{p}} + \boldsymbol{U}^{\text{n}})^* =$$
$$= U^{\text{p}2} + U^{\text{n}2} + 2\text{Re}\{\boldsymbol{U}^{\text{p}*}\boldsymbol{U}^{\text{n}}\} \tag{33}$$

$$U_{\text{S}}^2 = \boldsymbol{U}_{\text{S}}\boldsymbol{U}_{\text{S}}^* = (\alpha^*\boldsymbol{U}^{\text{p}} + \alpha\boldsymbol{U}^{\text{n}})(\alpha^*\boldsymbol{U}^{\text{p}} + \alpha\boldsymbol{U}^{\text{n}})^* =$$
$$= U^{\text{p}2} + U^{\text{n}2} + 2\text{Re}\{\alpha^*\boldsymbol{U}^{\text{p}*}\boldsymbol{U}^{\text{n}}\}. \tag{34}$$

Similarly

$$U_{\text{T}}^2 = U^{\text{p}2} + U^{\text{n}2} + 2\text{Re}\{\alpha\boldsymbol{U}^{\text{p}*}\boldsymbol{U}^{\text{n}}\}\ . \tag{35}$$

Also observe that

$$||\boldsymbol{u}||^2 = U_{\text{R}}^2 + U_{\text{S}}^2 + U_{\text{T}}^2 = 3(U^{\text{p}2} + U^{\text{n}2})\ . \tag{36}$$

The crms values of the supply voltage symmetrical components $\boldsymbol{U}^{\text{p}}$ and $\boldsymbol{U}^{\text{n}}$ have in general the form

$$\boldsymbol{U}^{\text{p}} = U^{\text{p}}\,e^{j\phi}\,,\qquad \boldsymbol{U}^{\text{n}} = U^{\text{n}}\,e^{j\varphi}$$

therefore, if we denote

$$\boldsymbol{U}^{\text{p}*}\boldsymbol{U}^{\text{n}} = U^{\text{p}}U^{\text{n}}e^{j(\varphi-\phi)} \stackrel{\text{df}}{=} \boldsymbol{W} = We^{j\psi} \tag{37}$$

admittance $\boldsymbol{Y}_{\text{d}}$, given by (31), can be expressed as

$$\boldsymbol{Y}_{\text{d}} = 2\,\frac{\boldsymbol{Y}_{\text{ST}}\text{Re}\{\boldsymbol{W}\} + \boldsymbol{Y}_{\text{TR}}\text{Re}\{\alpha^*\boldsymbol{W}\} + \boldsymbol{Y}_{\text{RS}}\text{Re}\{\alpha\boldsymbol{W}\}}{U^{\text{p}2} + U^{\text{n}2}}. \tag{38}$$

When the supply voltage asymmetry is specified by a complex asymmetry coefficient $\boldsymbol{a}$,

$$\frac{\boldsymbol{U}^{\text{n}}}{\boldsymbol{U}^{\text{p}}} = \frac{U^{\text{n}}e^{j\varphi}}{U^{\text{p}}e^{j\phi}} = \frac{U^{\text{n}}}{U^{\text{p}}}e^{j(\varphi-\phi)} \stackrel{\text{df}}{=} \boldsymbol{a} = ae^{j\psi} \tag{39}$$

then

$$\frac{\text{Re}\{\boldsymbol{W}\}}{U^{\text{p}2} + U^{\text{n}2}} = \frac{U^{\text{p}}U^{\text{n}}}{U^{\text{p}2} + U^{\text{n}2}}\text{Re}\{e^{j(\varphi-\phi)}\} =$$
$$= \frac{a}{1+a^2}\cos\psi \stackrel{\text{df}}{=} b\cdot\cos\psi \tag{40}$$

and the asymmetry dependent unbalanced admittance $\boldsymbol{Y}_{\text{d}n}$ can be rearranged, with restored index "n", to the form

$$\boldsymbol{Y}_{\text{d}n} = b_n[\boldsymbol{Y}_{\text{ST}n}\cos\psi_n + \boldsymbol{Y}_{\text{TR}n}\cos(\psi_n - \frac{2\pi}{3}) +$$
$$+ \boldsymbol{Y}_{\text{RS}n}\cos(\psi_n + \frac{2\pi}{3})]. \tag{41}$$

This formula reveals the dependence of the load unbalanced admittance $\boldsymbol{Y}_{\mathrm{d}n}$ on the load voltage asymmetry. It is zero both for balanced loads and zero for any loads supplied with symmetrical voltage harmonics.

Having the dependence of the admittance $\boldsymbol{Y}_{\mathrm{d}n}$ on the circuit parameters, also the unbalanced current three-phase rms value $||\boldsymbol{i}_{\mathrm{u}}||$ can be calculated not only from formula (14), but also in terms of the load admittances. Observe that the vector $\boldsymbol{I}_{\mathrm{u}n}$ of crms values (28) of the unbalanced current of the n^{th} order harmonic can be decomposed into vectors of the positive and negative sequence

$$\boldsymbol{I}_{\mathrm{u}n} = \boldsymbol{I}_{\mathrm{u}n}^{\mathrm{p}} + \boldsymbol{I}_{\mathrm{u}n}^{\mathrm{n}} \tag{42}$$

where

$$\boldsymbol{I}_{\mathrm{u}n}^{\mathrm{p}} = \mathbf{1}^{\mathrm{p}}(\boldsymbol{Y}_{\mathrm{d}n}\boldsymbol{U}_n^{\mathrm{p}} + \boldsymbol{Y}_{\mathrm{u}n}^{\mathrm{n}}\boldsymbol{U}_n^{\mathrm{n}}) \tag{43}$$

$$\boldsymbol{I}_{\mathrm{u}n}^{\mathrm{n}} = \mathbf{1}^{\mathrm{n}}(\boldsymbol{Y}_{\mathrm{d}n}\boldsymbol{U}_n^{\mathrm{n}} + \boldsymbol{Y}_{\mathrm{u}n}^{\mathrm{p}}\boldsymbol{U}^{\mathrm{p}})\,. \tag{44}$$

Thus the unbalanced current can be expressed as

$$\boldsymbol{i}_{\mathrm{u}} \overset{\mathrm{df}}{=} \sum_{n\in N}\boldsymbol{i}_{\mathrm{u}n} = \sqrt{2}\,\mathrm{Re}\sum_{n\in N}\boldsymbol{I}_{\mathrm{u}n}e^{jn\omega_1 t} = \boldsymbol{i}_{\mathrm{u}}^{\mathrm{p}} + \boldsymbol{i}_{\mathrm{u}}^{\mathrm{n}} \tag{45}$$

where

$$\boldsymbol{i}_{\mathrm{u}}^{\mathrm{p}} \overset{\mathrm{df}}{=} \sqrt{2}\,\mathrm{Re}\sum_{n\in N}(\boldsymbol{Y}_{\mathrm{d}n}\boldsymbol{U}_n^{\mathrm{p}} + \boldsymbol{Y}_{\mathrm{u}n}^{\mathrm{n}}\boldsymbol{U}_n^{\mathrm{n}})\mathbf{1}^{\mathrm{p}}e^{jn\omega_1 t} \tag{46}$$

$$\boldsymbol{i}_{\mathrm{u}}^{\mathrm{n}} \overset{\mathrm{df}}{=} \sqrt{2}\,\mathrm{Re}\sum_{n\in N}(\boldsymbol{Y}_{\mathrm{d}n}\boldsymbol{U}_n^{\mathrm{n}} + \boldsymbol{Y}_{\mathrm{u}n}^{\mathrm{p}}\boldsymbol{U}^{\mathrm{p}})\mathbf{1}^{\mathrm{n}}e^{jn\omega_1 t}. \tag{47}$$

These two components are symmetrical and of opposite sequence, so that they are mutually orthogonal, thus the three-phase rms value of the unbalanced current is

$$||\boldsymbol{i}_{\mathrm{u}}||^2 = ||\boldsymbol{i}_{\mathrm{u}}^{\mathrm{p}}||^2 + ||\boldsymbol{i}_{\mathrm{u}}^{\mathrm{n}}||^2 \tag{48}$$

where

$$||\boldsymbol{i}_{\mathrm{u}}^{\mathrm{p}}|| = \sqrt{3}\sqrt{\sum_{n\in N}|\boldsymbol{Y}_{\mathrm{d}n}\boldsymbol{U}_n^{\mathrm{p}} + \boldsymbol{Y}_{\mathrm{u}n}^{\mathrm{n}}\boldsymbol{U}_n^{\mathrm{n}}|^2} \tag{49}$$

$$||\boldsymbol{i}_{\mathrm{u}}^{\mathrm{n}}|| = \sqrt{3}\sqrt{\sum_{n\in N}|\boldsymbol{Y}_{\mathrm{d}n}\boldsymbol{U}_n^{\mathrm{n}} + \boldsymbol{Y}_{\mathrm{u}n}^{\mathrm{p}}\boldsymbol{U}^{\mathrm{p}}|^2} \tag{50}$$

thus, they are specified in terms of the load unbalanced admittances for harmonic frequencies of the positive and the negative sequence.

IV. Numerical Verification

The presented current decomposition into CPC is verified numerically with the circuit in Fig. 2. It was obtained without any restrictions as to the level of the supply voltage asymmetry, its distortion, and the load imbalance. To demonstrate that it is valid independently of the supply voltage asymmetry and distortion, and independently of the load imbalance, very high level of voltage distortion and asymmetry was assumed for this verification. It was assumed that the internal voltage $\boldsymbol{\varepsilon}$ fundamental harmonic of the supply system is E_{R1} = 100V, while the higher order harmonics, of the order n = 3, 5 and 7 have the same, very high rms value, equal to $E_{\mathrm{R3}}= E_{\mathrm{R5}}= E_{\mathrm{R7}}=$ 20V. It was also assumed, that the voltage at the supply terminal T is $E_{\mathrm{T}} = E_{\mathrm{R}}/2$. Moreover, the load voltage asymmetry was increased by an extra phase shift of voltages at line S and T, namely, $\Delta\varphi_{\mathrm{S}}$ = 10 deg. and $\Delta\varphi_{\mathrm{T}} = -10$ deg., the same for each voltage harmonic. Such assumptions, by making the system asymmetry significant, enables reliable numerical verification and can enhance the credibility of the developed current decomposition. The load parameters for the fundamental harmonic are assumed to be equal to

$$R_{\mathrm{R}} = R_{\mathrm{T}} = 1.0\ \Omega,\quad X_{\mathrm{R}} = X_{\mathrm{T}} = 1.0\ \Omega,\quad B_{\mathrm{R}} = B_{\mathrm{T}} = 0.50\ \mathrm{S}.$$

The load is supplied from an ideal transformer in Δ/Y configuration with the turn ratio $\sqrt{3}:1$.

The crms values of the zero sequence symmetrical component of the supply voltage harmonics are equal to

$$\boldsymbol{U}_1^{\mathrm{z}} = 22.56\,e^{-j44.0^{\mathrm{o}}}\,\mathrm{V},\quad \boldsymbol{U}_3^{\mathrm{z}} = 16.52\,e^{j2.0^{\mathrm{o}}}\,\mathrm{V},$$

$$\boldsymbol{U}_5^{\mathrm{z}} = 2.56\,e^{j84.6^{\mathrm{o}}}\,\mathrm{V},\quad \boldsymbol{U}_7^{\mathrm{z}} = 4.51\,e^{-j44.0^{\mathrm{o}}}\,\mathrm{V}.$$

The crms values of the positive and the negative sequence symmetrical components of load voltage harmonics as referenced to the artificial zero are

$$\boldsymbol{U}_1^{\mathrm{p}} = 82.62\,e^{j2.0^{\mathrm{o}}}\,\mathrm{V},\quad \boldsymbol{U}_3^{\mathrm{p}} = 2.56\,e^{j84.7^{\mathrm{o}}}\,\mathrm{V},$$

$$\boldsymbol{U}_5^{\mathrm{p}} = 4.51\,e^{-j44.0^{\mathrm{o}}}\,\mathrm{V},\quad \boldsymbol{U}_7^{\mathrm{p}} = 16.52\,e^{j2.0^{\mathrm{o}}}\,\mathrm{V}.$$

$$\boldsymbol{U}_1^{\mathrm{n}} = 12.82\,e^{j84.7^{\mathrm{o}}}\,\mathrm{V},\quad \boldsymbol{U}_3^{\mathrm{n}} = 4.51\,e^{-j44.0^{\mathrm{o}}}\,\mathrm{V},$$

$$\boldsymbol{U}_5^{\mathrm{n}} = 16.52\,e^{j2.0^{\mathrm{o}}}\,\mathrm{V},\quad \boldsymbol{U}_7^{\mathrm{n}} = 2.56\,e^{j84.6^{\mathrm{o}}}\,\mathrm{V}.$$

The crms values of harmonics of line current are

$$\boldsymbol{I}_1 = \begin{bmatrix}\boldsymbol{I}_{\mathrm{R1}}\\ \boldsymbol{I}_{\mathrm{S1}}\\ \boldsymbol{I}_{\mathrm{T1}}\end{bmatrix} = \begin{bmatrix}127.8\,e^{j10.6^{\mathrm{o}}}\\ 81.9\,e^{-j145.0^{\mathrm{o}}}\\ 63.1\,e^{j158.1^{\mathrm{o}}}\end{bmatrix}\mathrm{A},\quad \boldsymbol{I}_3 = \begin{bmatrix}\boldsymbol{I}_{\mathrm{R3}}\\ \boldsymbol{I}_{\mathrm{S3}}\\ \boldsymbol{I}_{\mathrm{T3}}\end{bmatrix} = \begin{bmatrix}12.8\,e^{j75.8^{\mathrm{o}}}\\ 4.2\,e^{-j179.8^{\mathrm{o}}}\\ 12.4\,e^{-j85.1^{\mathrm{o}}}\end{bmatrix}\mathrm{A}$$

$$\boldsymbol{I}_5 = \begin{bmatrix}\boldsymbol{I}_{\mathrm{R5}}\\ \boldsymbol{I}_{\mathrm{S5}}\\ \boldsymbol{I}_{\mathrm{T5}}\end{bmatrix} = \begin{bmatrix}138.0\,e^{j81.7^{\mathrm{o}}}\\ 83.7\,e^{-j115.9^{\mathrm{o}}}\\ 63.5\,e^{-j74.8^{\mathrm{o}}}\end{bmatrix}\mathrm{A},\quad \boldsymbol{I}_7 = \begin{bmatrix}\boldsymbol{I}_{\mathrm{R7}}\\ \boldsymbol{I}_{\mathrm{S7}}\\ \boldsymbol{I}_{\mathrm{T7}}\end{bmatrix} = \begin{bmatrix}171.8\,e^{j100.2^{\mathrm{o}}}\\ 110.1\,e^{-j55.3^{\mathrm{o}}}\\ 85.0\,e^{-j112.2^{\mathrm{o}}}\end{bmatrix}\mathrm{A}.$$

The rms values of the line-to-artificial zero voltages and line currents in the circuit are shown in Fig. 2.

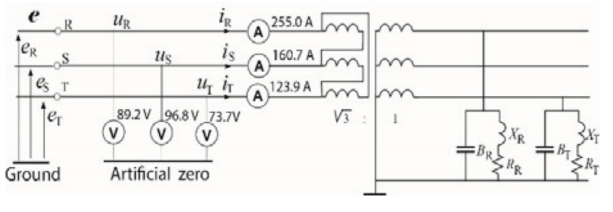

Fig. 2. Rms values of the line-to-artificial zero voltages and load line currents.

Results of the circuit analysis with respect to the load equivalent parameters for harmonics and harmonic active and reactive powers are compiled in Table.1.

	n	1	3	5	7
P_n	W	21380	12	80	34
Q_n	VAr	0	−142	−4780	−5747
$\lVert\boldsymbol{u}_n\rVert$	V	144.80	8.9	29.7	29.0
$G_{\mathrm{b}n}$	S	1.0194	0.1464	0.0905	0.0408
$B_{\mathrm{b}n}$	S	0	1.7562	5.4302	6.8503

Table 1. Results of the circuit analysis.

The load unbalanced admittances for the positive sequence are equal to

$$\boldsymbol{Y}_{\mathrm{u1}}^{\mathrm{p}} = 0.500\,e^{j0^{\mathrm{o}}}\,\mathrm{S},\quad \boldsymbol{Y}_{\mathrm{u3}}^{\mathrm{p}} = 1.204\,e^{j85.2^{\mathrm{o}}}\,\mathrm{S},$$

$$\boldsymbol{Y}_{\mathrm{u5}}^{\mathrm{p}} = 2.308\,e^{j89.0^{\mathrm{o}}}\,\mathrm{S},\quad \boldsymbol{Y}_{\mathrm{u7}}^{\mathrm{p}} = 3.360\,e^{j89.7^{\mathrm{o}}}\,\mathrm{S}.$$

Observe, that for the selected load $\boldsymbol{Y}_{\mathrm{u}n}^{\mathrm{n}} = \boldsymbol{Y}_{\mathrm{u}n}^{\mathrm{p}}$.

The active power of the load is P = 21506 W and the three-phase rms value of the supply voltage as referenced to the artificial zero is $\|\boldsymbol{u}\|$ = 150.91V, so that, the equivalent balanced conductance is $G_{\rm b} = P/\|\boldsymbol{u}\|^2 = 0.9443$ S .

The voltage asymmetry dependent admittances for harmonics are equal to

$$Y_{\rm d1}^{\rm p} = 0.0194\, e^{j180^{\rm o}}\,{\rm S}, \qquad Y_{\rm u3}^{\rm p} = 0.6460\, e^{j85.2^{\rm o}}\,{\rm S},$$

$$Y_{\rm u5}^{\rm p} = 0.8149\, e^{-j90.0^{\rm o}}\,{\rm S}, \qquad Y_{\rm u7}^{\rm p} = 0.1303\, e^{-j90.3^{\rm o}}\,{\rm S}.$$

Having values of equivalent parameters of the load as compiled in Table 1, three-phase rms values of CPC of the load current can be calculated, namely

$$\|\boldsymbol{i}_{\rm a}\| = G_{\rm b}\|\boldsymbol{u}\| = 142.5\ {\rm A}$$

$$\|\boldsymbol{i}_{\rm r}\| = \sqrt{\sum_{n\in N} B_{{\rm b}n}^2\, \|\boldsymbol{u}_n\|^2} = 256.0\ {\rm A}$$

$$\|\boldsymbol{i}_{\rm s}\| = \sqrt{\sum_{n\in N} (G_{{\rm b}n} - G_{\rm b})^2\, \|\boldsymbol{u}_n\|^2} = 38.7\ {\rm A}$$

$$\|\boldsymbol{i}_{\rm u}^{\rm p}\| = 65.0\ {\rm A}\ , \qquad \|\boldsymbol{i}_{\rm u}^{\rm n}\| = 121.0\ {\rm A}$$

so that the rms value of the unbalanced current is

$$\|\boldsymbol{i}_{\rm u}\| = \sqrt{\|\boldsymbol{i}_{\rm u}^{\rm p}\|^2 + \|\boldsymbol{i}_{\rm u}^{\rm n}\|^2} = 137.4\ {\rm A}.$$

The root of the sum of squares of three-phase rms values of the CPC has to be equal to the three-phase rms value of the load current which is equal to

$$\|\boldsymbol{i}\| = \sqrt{\|i_{\rm R}\|^2 + \|i_{\rm S}\|^2 + \|i_{\rm T}\|^2} = \sqrt{255.0^2 + 160.7^2 + 123.9^2} = 325.9\ {\rm A}\ .$$

Indeed,

$$\|\boldsymbol{i}\| = \sqrt{\|\boldsymbol{i}_{\rm a}\|^2 + \|\boldsymbol{i}_{\rm r}\|^2 + \|\boldsymbol{i}_{\rm s}\|^2 + \|\boldsymbol{i}_{\rm u}\|^2} =$$

$$= \sqrt{142.5^2 + 256.0^2 + 38.7^2 + 137.4^2} = 325.9\ {\rm A}$$

with a numerical error on the level of 10^{-7}. These numerical results confirm the correctness of the load current decomposition into the Currents' Physical Components. Thus, the CPC-based power theory enables us to associate distinctive physical phenomena in the load with specific components of the load current and express them in terms of the load parameters, the supply voltage harmonics, and asymmetry.

V. PARAMETERS OF A BALANCING COMPENSATOR

The power factor λ can be expressed in terms of three-phase rms values of the Currents' Physical Components as follows

$$\lambda = \frac{P}{S} = \frac{\|\boldsymbol{i}_{\rm a}\|}{\|\boldsymbol{i}\|} = \frac{\|\boldsymbol{i}_{\rm a}\|}{\sqrt{\|\boldsymbol{i}_{\rm a}\|^2 + \|\boldsymbol{i}_{\rm r}\|^2 + \|\boldsymbol{i}_{\rm s}\|^2 + \|\boldsymbol{i}_{\rm u}\|^2}} \tag{51}$$

thus the presence of the reactive, scattered and unbalanced currents reduces the power factor value. Their reduction contributes to its increase.

The power factor can be improved using switching compensators or using reactive compensators. This paper is confined entirely to a method of calculation of LC parameters of reactive balancing compensators, however.

A reactive balancing compensator is composed of three one-ports configured in Δ, shown in Fig. 3, although Y structure could be used as well.

Since all CPC are sums of harmonics, compensation can be analyzed by a harmonic-by-harmonic approach. Therefore, for any harmonic of the $n^{\rm th}$ order the load with a compensator can be presented as shown in Fig. 3. The load can be specified in terms of four admittances for harmonics: $Y_{{\rm b}n}$, $Y_{{\rm u}n}^{\rm p}$, $Y_{{\rm u}n}^{\rm n}$ and $Y_{{\rm d}n}$. Such a compensator changes the vector of the load line current harmonic from $\boldsymbol{i}_n$ to $\boldsymbol{i}_n$'.

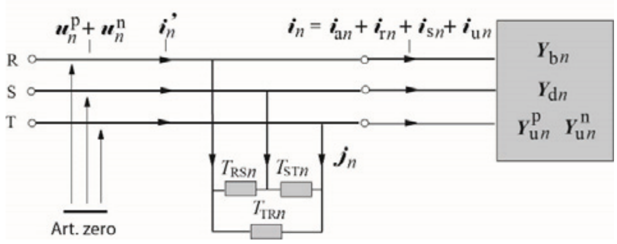

Fig. 3. Equivalent admittances of the load and compensator for the n^{th} order harmonic.

It will be assumed that the compensator is a lossless device and the load is supplied from an ideal voltage source, i.e., with zero internal impedance. At such assumptions the compensator does not affect the voltage and the active power at the supply source terminals. Consequently, the balanced conductance $G_{\rm b}$ and conductances $G_{{\rm b}n}$ for harmonics remain unchanged, which means that the compensator cannot affect the scattered current $\boldsymbol{i}_{\rm s}$. Thus, in the presence of the scattered current the unity power factor cannot be achieved by a reactive compensator. Only the reactive and unbalanced currents can be compensated.

Since the line-to-line admittances of the compensator for harmonic frequencies are

$$Y_{{\rm ST}n} = jT_{{\rm ST}n}\,, \quad Y_{{\rm TR}n} = jT_{{\rm TR}n}\,, \quad Y_{{\rm RS}n} = jT_{{\rm RS}n}$$

the compensator can be specified for harmonic frequencies in terms of the balanced susceptance

$$B_{{\rm Cb}n} = \frac{T_{{\rm ST}n}U_{{\rm ST}n}^2 + T_{{\rm TR}n}U_{{\rm TR}n}^2 + T_{{\rm RS}n}U_{{\rm RS}n}^2}{\|\boldsymbol{u}_n\|^2} \tag{52}$$

and unbalanced admittances

$$Y_{{\rm Cu}n}^{\rm p} = -j\,(T_{{\rm ST}n} + \alpha\, T_{{\rm TR}n} + \alpha^*\, T_{{\rm RS}n}) \tag{53}$$

$$Y_{{\rm Cu}n}^{\rm n} = -j\,(T_{{\rm ST}n} + \alpha^*T_{{\rm TR}n} + \alpha\, T_{{\rm RS}n}) \tag{54}$$

$$Y_{{\rm Cd}n} = jb_n[T_{{\rm ST}n}\cos\psi_n + T_{{\rm TR}n}\cos(\psi_n - \tfrac{2\pi}{3}) + T_{{\rm RS}n}\cos(\psi_n + \tfrac{2\pi}{3})]. \tag{55}$$

The compensator reduces entirely the reactive current harmonic of the $n^{\rm th}$ order if its susceptance $B_{{\rm Cb}n}$ satisfies the condition

$$B_{{\rm Cb}n} + B_{{\rm b}n} = 0\,. \tag{56}$$

Taking (31) into account, the $n^{\rm th}$ order harmonic of the unbalanced current $\boldsymbol{i}_{\rm u}$ in the reference line R of the load with the compensator is equal to

$$\boldsymbol{I}_{{\rm Ru}n} = (\boldsymbol{Y}_{{\rm Cd}n} + \boldsymbol{Y}_{{\rm d}n})\boldsymbol{U}_{{\rm R}n} + (\boldsymbol{Y}_{{\rm Cu}n}^{\rm p} + \boldsymbol{Y}_{{\rm u}n}^{\rm p})\boldsymbol{U}_n^{\rm p} + (\boldsymbol{Y}_{{\rm Cu}n}^{\rm n} + \boldsymbol{Y}_{{\rm u}n}^{\rm n})\boldsymbol{U}_n^{\rm n}\,. \tag{57}$$

This harmonic of the unbalanced current is compensated entirely on the condition that

$$(\boldsymbol{Y}_{{\rm Cd}n} + \boldsymbol{Y}_{{\rm d}n})\boldsymbol{U}_{{\rm R}n} + (\boldsymbol{Y}_{{\rm Cu}n}^{\rm p} + \boldsymbol{Y}_{{\rm u}n}^{\rm p})\boldsymbol{U}_n^{\rm p} + (\boldsymbol{Y}_{{\rm Cu}n}^{\rm n} + \boldsymbol{Y}_{{\rm u}n}^{\rm n})\boldsymbol{U}_n^{\rm n} = 0\,. \tag{58}$$

It can be satisfied if condition (58) is satisfied, separately, for the real and for the imaginary part. In such a way three equations, which are needed for calculating three susceptances $T_{\mathrm{RS}n}$, $T_{\mathrm{ST}n}$ and $T_{\mathrm{TR}n}$ are obtained.

On the condition that $\boldsymbol{U}_n^{\mathrm{p}} \neq 0$, eqn. (58) can be modified to

$$(\boldsymbol{Y}_{\mathrm{Cd}n}+\boldsymbol{Y}_{\mathrm{d}n})(1+\boldsymbol{a}_n)+(\boldsymbol{Y}_{\mathrm{Cu}n}^{\mathrm{p}}+\boldsymbol{Y}_{\mathrm{u}n}^{\mathrm{p}})+(\boldsymbol{Y}_{\mathrm{Cu}n}^{\mathrm{n}}+\boldsymbol{Y}_{\mathrm{u}n}^{\mathrm{n}})\boldsymbol{a}_n = 0 \tag{59}$$

with the complex coefficient $\boldsymbol{a}_n$, defined by (39). In this equation, the unbalanced admittances of the compensator are specified by formulae (53-55). Assuming that

$$\boldsymbol{c}_{1n} \stackrel{\mathrm{df}}{=} jb_n\cos\psi_n, \qquad \boldsymbol{c}_{2n} \stackrel{\mathrm{df}}{=} jb_n\cos(\psi_n-\frac{2\pi}{3}),$$
$$\boldsymbol{c}_{3n} \stackrel{\mathrm{df}}{=} jb_n\cos(\psi_n+\frac{2\pi}{3}) \tag{60}$$

the unbalanced admittance $\boldsymbol{Y}_{\mathrm{Cd}n}$ can be expressed in the form

$$\boldsymbol{Y}_{\mathrm{Cd}n} = \boldsymbol{c}_{1n}T_{\mathrm{ST}n} + \boldsymbol{c}_{2n}T_{\mathrm{TR}n} + \boldsymbol{c}_{3n}T_{\mathrm{RS}n} \tag{61}$$

and equation (58) can be rearranged to a linear form of the compensator susceptances

$$\boldsymbol{A}_n T_{\mathrm{ST}n} + \boldsymbol{B}_n T_{\mathrm{TR}n} + \boldsymbol{C}_n T_{\mathrm{RS}n} + \boldsymbol{D}_n = 0\,. \tag{62}$$

When we denote

$$\boldsymbol{d}_n \stackrel{\mathrm{df}}{=} \boldsymbol{a}_n + 1\,, \qquad \boldsymbol{f}_n \stackrel{\mathrm{df}}{=} \alpha^* + \alpha\,\boldsymbol{a}_n\,, \qquad \boldsymbol{g}_n \stackrel{\mathrm{df}}{=} \alpha + \alpha^*\boldsymbol{a}_n \tag{63}$$

then the coefficients in the form (61) are equal to

$$\boldsymbol{A}_n = (\boldsymbol{c}_{1n} - j1)\,\boldsymbol{d}_n\,, \qquad \boldsymbol{B}_n = \boldsymbol{c}_{2n}\boldsymbol{d}_n - j\cdot\boldsymbol{g}_n \tag{64}$$

$$\boldsymbol{C}_n = \boldsymbol{c}_{3n}\boldsymbol{d}_n - j\cdot\boldsymbol{f}_n\,, \qquad \boldsymbol{D}_n = \boldsymbol{Y}_{\mathrm{u}n}^{\mathrm{n}}\boldsymbol{a}_n + \boldsymbol{Y}_{\mathrm{d}n}\boldsymbol{d}_n + \boldsymbol{Y}_{\mathrm{u}n}^{\mathrm{p}}. \tag{65}$$

The condition (56) for the n^{th} order harmonic can be rearranged to the form

$$U_{\mathrm{ST}n}^2 T_{\mathrm{ST}n} + U_{\mathrm{TR}n}^2 T_{\mathrm{TR}n} + U_{\mathrm{RS}n}^2 T_{\mathrm{RS}n} = -B_{\mathrm{b}n}\,||\boldsymbol{u}_n||^2\,. \tag{66}$$

The linear form (58) has to be satisfied for the real and for the imaginary part of it, thus

$$\mathrm{Re}\{\boldsymbol{A}_n T_{\mathrm{ST}n} + \boldsymbol{B}_n T_{\mathrm{TR}n} + \boldsymbol{C}_n T_{\mathrm{RS}n} + \boldsymbol{D}_n\} = 0 \tag{67}$$

$$\mathrm{Im}\{\boldsymbol{A}_n T_{\mathrm{ST}n} + \boldsymbol{B}_n T_{\mathrm{TR}n} + \boldsymbol{C}_n T_{\mathrm{RS}n} + \boldsymbol{D}_n\} = 0\,. \tag{68}$$

Equations (66-68) specify the compensator's line-to-line susceptances for the n^{th} order harmonic. They can be written the form

$$\begin{bmatrix} U_{\mathrm{ST}n}^2, & U_{\mathrm{TR}n}^2, & U_{\mathrm{RS}n}^2 \\ \mathrm{Re}\boldsymbol{A}_n, & \mathrm{Re}\boldsymbol{B}_n, & \mathrm{Re}\,\boldsymbol{C}_n \\ \mathrm{Im}\boldsymbol{A}_n, & \mathrm{Im}\,\boldsymbol{B}_n, & \mathrm{Im}\,\boldsymbol{C}_n \end{bmatrix} \begin{bmatrix} T_{\mathrm{ST}n} \\ T_{\mathrm{TR}n} \\ T_{\mathrm{RS}n} \end{bmatrix} = \begin{bmatrix} -B_{\mathrm{b}n}\,||\boldsymbol{u}_n||^2 \\ -\mathrm{Re}\,\boldsymbol{D}_n \\ -\mathrm{Im}\,\boldsymbol{D}_n \end{bmatrix}. \tag{69}$$

Numerical Illustration 1. For the circuit used in Section IV for numerical verification with load parameters compiled in Table 1, the compensator equation (69) results in susceptances of the compensator's branches compiled in Table 2.

	n	1	3	5	7
$T_{\mathrm{ST}n}$	S	– 0.0567	0.0032	0.0093	– 0.0023
$T_{\mathrm{TR}n}$	S	– 0.3530	–1.187	–2.282	–3.374
$T_{\mathrm{TR}n}$	S	0.2519	–1.348	–2.327	–3.350

Table 2. Compensator's line-to-line susceptances.

The results of compensation are shown in Fig. 4.

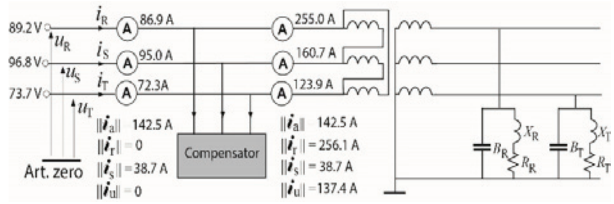

Fig. 4. Results of compensation of the reactive and unbalanced currents.

It can be observed that despite of total compensation of the unbalanced and reactive currents, the vector of the supply current is not proportional to the load voltage vector. It is because the supply current still contains the scattered current $\boldsymbol{\imath}_{\mathrm{s}}$ which is not proportional to the voltage vector. Only the active current $\boldsymbol{\imath}_{\mathrm{a}}$ is proportional to that vector.

VI. Reactive Compensators

A reactive compensator for the reactive and the unbalanced currents compensation can be composed of three one-ports connected as shown in Fig. 3, with susceptances for harmonic frequencies $T_{\mathrm{ST}n}$, $T_{\mathrm{TR}n}$ and $T_{\mathrm{RS}n}$, that satisfy equation (69).

Methods of selection the one-port structure and calculation of its LC parameters in such a way that for harmonic frequencies such a one-port has required susceptances T_n, are well developed in a frame of network synthesis [2]. Only a few conclusions from the network synthesis are used in this paper to draft an approach to a reactive compensator synthesis. This draft is illustrated with synthesis of a compensator branch with susceptance T_n specified, to reduce amount of calculation, for only three harmonics.

Susceptance $T(\omega) = \mathrm{Im}\{Y(j\omega)\}$ of a reactive one-port is an increasing function [2] of frequency, i.e.,

$$\frac{d}{d\omega}T(\omega) > 0 \tag{70}$$

which changes its value from $+\infty$ to $-\infty$ at the function POLEs, p_1, p_2….as shown on an example, in Fig. 5. Susceptance T_n for harmonic frequency is the value of the function $T(\omega)$ for frequency $\omega = n\omega_1$.

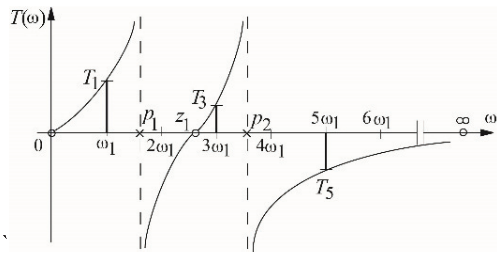

Fig. 5. Example of change of susceptance $T(\omega)$ with frequency.

When the set N of the voltage harmonic orders has M elements, then the susceptance $T(\omega)$ can be specified at these M harmonic frequencies. In the example shown in Fig. 5, the set of harmonics $N = \{1, 3, 5\}$, meaning $M = 3$, the branch should have susceptance T_1, T_3 and T_5.

The admittance of a reactive one-port is a function of the complex variable s. In the case illustrated in Fig. 5 it has a general form

$$Y(s) = A\,s\,\frac{(s^2+z_1^2)}{(s^2+p_1^2)(s^2+p_2^2)} \tag{71}$$

i.e., it is determined by parameters: A, z_1, p_1, and p_2.

The minimum number K of reactive elements needed for synthesis of a one-port is specified by the sum of the number of POLEs, N_p, and the number of ZEROs, N_z, not located at $\omega = 0$ or $\omega = \infty$, on the positive axis of frequency, ω, namely

$$K = N_p + N_z + 1\,. \tag{72}$$

For example, the admittance specified by (71) has two POLEs, so that $N_p = 2$, one ZERO, thus $N_z = 1$, and consequently, $K = 4$, i.e., four reactive elements are needed.

The number of POLEs and ZEROs depends on the sign pattern of susceptances T_n. When two neighboring susceptances have the same sign, they have to be separated by a POLE and a ZERO. When they have an opposite sign, they have to be separated by a POLE or a ZERO.

On the imaginary axis of the complex plane s, i.e., $s = j\omega$, the admittance function in this illustration has the form

$$Y(j\omega) = A\,j\omega\,\frac{(-\omega^2+z_1^2)}{(-\omega^2+p_1^2)(-\omega^2+p_2^2)} = j\,T(\omega) \tag{73}$$

i.e., its imaginary part represents the one-port susceptance $T(\omega)$. Its value for frequencies $\omega = n\omega_1$, assuming that the fundamental frequency is normalized to $\omega_1 = 1$ rad/s,

$$T(n\omega_1) = T(n) \stackrel{\text{df}}{=} T_n = A\,n\,\frac{(z_1^2-n^2)}{(p_1^2-n^2)(p_2^2-n^2)}\,. \tag{74}$$

The susceptance T_n in this equation is known, parameters A, z_1, p_1 and p_2 are unknown. Thus, the susceptances T_n of the one-port for harmonic frequencies are specified by four parameters: A, z_1, p_1, and p_2. There are only three constraints for admittance $Y(s)$, i.e., its values T_1, T_3, and T_5 in this illustration. Therefore, one parameter can be selected at a designer discretion. Thus, there is an infinite number of admittances specified by (71).

Let T_1= 2.0 S, T_3= 1.0 S and T_5.= – 0.8 S. and $z_1^2 = 4$. At such assumption, three equations developed from (74) for $n = 1$, $n = 3$, $n = 5$, have the form

$$\begin{aligned} &A\cdot 1\,\frac{(4-1)}{(p_1^2-1)(p_2^2-1)} = 2.0\\ &A\cdot 3\,\frac{(4-9)}{(p_1^2-9)(p_2^2-9)} = 1.0\\ &A\cdot 5\,\frac{(4-25)}{(p_1^2-25)(p_2^2-25)} = -0.8. \end{aligned} \tag{75}$$

Their solution with respect to parameters A, p_1 and p_2 result in

$$A = 2.162, \quad p_1^2 = 1.266, \quad p_2^2 = 13.192$$

hence the admittance of the compensator branch is

$$Y(s) = 2.162\,s\,\frac{(s^2+4)}{(s^2+1.266)(s^2+13.192)}\,. \tag{76}$$

This admittance function can be rearranged to the form

$$Y(s) = \frac{1.667\,s}{s^2+1.266} + \frac{0.495\,s}{s^2+13.192}\,. \tag{77}$$

which stands for admittance of a one-port composed of two LC branches connected in parallel as shown in Fig. 6

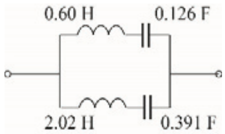

Fig. 6. Structure and parameters of a compensator branch.

This draft of the procedure of the compensator branches synthesis demonstrates that having values of the susceptance of a branch for harmonic frequencies T_n, its structure and LC parameters can be found. It shows that the compensator structure can be very complex, however. This complexity could be a major obstacle for compensation, especially of variable loads, when a compensator should have adaptive property. Therefore, the results on compensation obtained above have only cognitive rather than practical merits. They demonstrate that a reactive compensator can compensate entirely both the reactive and the unbalance currents of unbalanced loads supplied with nonsinusoidal and asymmetrical voltage. These results could have practical merits on the condition that the complexity of the compensator can be reduced. Complete compensation by a compensator with reduced complexity is, of course, not possible. Such a compensator can only reduce the reactive and unbalanced currents, but not compensate them entirely.

V. MINIMIZATION OF REACTIVE AND UNBALANCED CURRENTS

Let us suppose that there are two compensators configured in Δ, the first one, in Fig. 7a, is composed of branches with susceptances T_n that satisfy eqn. (69) and the other one, in Fig. 7b, with susceptances D_n.

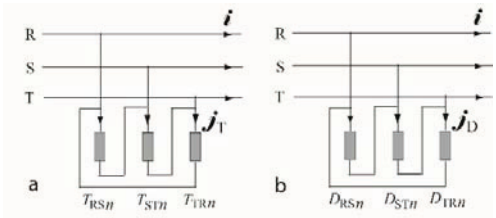

Fig. 7. Compensator composed of branches (a) with susceptance T_n and (b) with susceptance D_n.

The vector of branch currents in the compensator in Fig. 7a are

$$\boldsymbol{j}_T = \sqrt{2}\,\mathrm{Re}\sum_{n\in N} j\boldsymbol{T}_n \bullet \boldsymbol{U}_n\, e^{jn\omega_1 t} \tag{78}$$

where

$$\boldsymbol{T}_n \stackrel{\text{df}}{=} [T_{STn}, T_{TRn}, T_{RSn}]^{T} \tag{79}$$

is the and matrix of susceptances that satisfy eqn. (69) and

$$\boldsymbol{U}_n = [\boldsymbol{U}_{STn}, \boldsymbol{U}_{TRn}, \boldsymbol{U}_{RSn}]^{T}. \tag{80}$$

Let us assume that a compensator branch cannot be built of more reactive elements than two. Moreover, to avoid series resonance of the compensator with the supply source impedance, usually inductive, the compensator branch for higher order harmonics should be inductive. At such assumptions, a branch cannot be more complex than branches shown in Fig. 8.

Fig. 8. Acceptable branches of a compensator.

Susceptances of such branches for harmonics, denoted by D, are

$$D_n = -\frac{1}{n\omega_1 L} \quad \text{or} \quad D_n = \frac{n\omega_1 C}{1-n^2\omega_1^2 LC}\,. \tag{81}$$

The vector of branch currents of such a compensator is

$$\boldsymbol{j}_{\mathrm{D}} = \sqrt{2}\,\mathrm{Re}\sum_{n\in N} j\boldsymbol{D}_n \bullet \boldsymbol{U}_n\, e^{jn\omega_1 t} \tag{82}$$

where

$$\boldsymbol{D}_n \stackrel{\mathrm{df}}{=} \left[D_{\mathrm{ST}n},\, D_{\mathrm{TR}n},\, D_{\mathrm{RS}n}\right]^{\mathrm{T}}. \tag{83}$$

The distance, in terms of three-phase rms value, between the branch current of an ideal compensator and the compensator with reduced complexity is

$$d \stackrel{\mathrm{df}}{=} \|\, \boldsymbol{j}_{\mathrm{T}} - \boldsymbol{j}_{\mathrm{D}} \|\,. \tag{84}$$

The LC parameters of the compensator of the reduced complexity should be selected such that this distance is minimum.

The branch currents difference can be expressed as

$$\boldsymbol{j}_{\mathrm{T}} - \boldsymbol{j}_{\mathrm{D}} = \sqrt{2}\,\mathrm{Re}\sum_{n\in N} j[\boldsymbol{T}_n - \boldsymbol{D}_n]\bullet \boldsymbol{U}_n\, e^{jn\omega_1 t} = \sqrt{2}\,\mathrm{Re}\sum_{n\in N} \Delta\boldsymbol{J}_n\, e^{jn\omega_1 t} \tag{85}$$

where

$$\Delta\boldsymbol{J}_n = j\begin{bmatrix} T_{\mathrm{ST}n} - D_{\mathrm{ST}n} \\ T_{\mathrm{TR}n} - D_{\mathrm{TR}n} \\ T_{\mathrm{RS}n} - D_{\mathrm{RS}n} \end{bmatrix} \bullet \begin{bmatrix} \boldsymbol{U}_{\mathrm{ST}n} \\ \boldsymbol{U}_{\mathrm{TR}n} \\ \boldsymbol{U}_{\mathrm{RS}n} \end{bmatrix} \tag{86}$$

and consequently, the distance d of these current vectors is

$$d = \|\, \boldsymbol{j}_{\mathrm{T}} - \boldsymbol{j}_{\mathrm{D}} \| = \sqrt{\sum_{n\in N} \Delta\boldsymbol{J}_n^{\mathrm{T}} \Delta\boldsymbol{J}_n^{*}} = \sqrt{\sum_{n\in N}\sum_{k\in K} (T_{kn} - D_{kn})^2 U_{kn}^2} \tag{87}$$

where k is an index from the set $K = \{\mathrm{ST}, \mathrm{TR}, \mathrm{RS}\}$. The sequence of summation over harmonic orders n and the branch index k could be switched, so that

$$d = \sqrt{\sum_{k\in K}\sum_{n\in N} (T_{kn} - D_{kn})^2 U_{nk}^2} = \sqrt{\sum_{k\in K} d_k^2}\,. \tag{88}$$

Since the terms under the root are non-negative numbers, so that mutual cancellation is not possible, the distance d is minimum on the condition that all distances d_k for individual branches

$$d_k = \sqrt{\sum_{n\in N} (T_{kn} - D_{kn})^2 U_{kn}^2} \tag{89}$$

are minimum, i.e., for each branch of the compensator

$$\sum_{n\in N} (T_{kn} - D_{kn})^2 U_{kn}^2 = \mathrm{Min.} \tag{90}$$

Susceptances D_{kn}, dependent on LC parameters of the compensator branches, shown in Fig. 8, have to minimize condition (90).

The supply voltage rms value of the fundamental harmonic U_{k1} in distribution systems is much higher than the rms value of other harmonics. Because of that, the component of (90) with U_{k1} is the dominating one. Therefore, the term $(T_{k1} - D_{k1})$ should be as close to zero as possible. Thus, when $T_{k1} > 0$, the compensator branch should be chosen such that $D_{k1} < 0$, i.e., the purely inductive branch. When $T_{k1} < 0$, the compensator branch should be chosen such that $D_{k1} > 0$, i.e., the LC branch. Consequently, for purely inductive branches, the inductance L_k should be chosen such that

$$\sum_{n\in N} (T_{kn} + \frac{1}{n\omega_1 L_k})^2 U_{kn}^2 = \mathrm{Min.} \tag{91}$$

For LC branches, the inductance L_k and capacitance C_k should be chosen such that

$$\sum_{n\in N} (T_{kn} - \frac{n\omega_1 C_k}{1-n^2\omega_1^2 L_k C_k})^2 U_{kn}^2 = \mathrm{Min.} \tag{92}$$

The condition (92) is satisfied when its derivative with respect to L_k is zero, i.e.,

$$\frac{d}{dL_k}\{\sum_{n\in N} (T_{kn} + \frac{1}{n\omega_1 L_k})^2 U_{kn}^2\} = 0 \tag{93}$$

and this condition results in the optimum value of the branch inductance

$$L_{k,\,\mathrm{opt}} = -\sum_{n\in N}\frac{1}{n^2}U_{kn}^2 \times [\omega_1 \sum_{n\in N} T_{kn}\frac{1}{n}U_{kn}^2]^{-1}\,. \tag{94}$$

The term on the left side of (92) is a function of two variables, the inductance L_k and capacitance C_k. With respect to L_k it is continuously declining function, meaning it does not have a minimum for any finite value of inductance L_k. It has to be selected at a designer discretion. When it is selected, the capacitance C_k can be calculated such that (92) is minimum. When it is minimum, derivative of (92) with respect to C_k is zero, i.e.,

$$\frac{d}{dC_k}\{\sum_{n\in N} (T_{kn} - \frac{n\omega_1 C_k}{1-n^2\omega_1^2 L_k C_k})^2 U_{kn}^2\} = 0\,. \tag{95}$$

It results in the equation;

$$\sum_{n\in N}\frac{T_{kn}\, n U_{kn}^2}{(1-n^2\omega_1^2 L_k C_k)^2} - \sum_{n\in N}\frac{n^2\omega_1 C_k U_{kn}^2}{(1-n^2\omega_1^2 L_k C_k)^3} = 0 \tag{96}$$

which cannot be solved directly with respect to the optimum value of the capacitance C_k. Numerical methods are needed for that. In particular, it can be solved [7] in an iterative process

$$C_{k,s+1} = \sum_{n\in N}\frac{T_{kn}\, n U_{kn}^2}{(1-n^2\omega_1^2 L_k C_{k,s})^2} \times [\omega_1 \sum_{n\in N}\frac{n^2 U_{kn}^2}{(1-n^2\omega_1^2 L_k C_{k,s})^3}]^{-1} \tag{97}$$

which results in a sequence of values usually convergent to the optimum capacitance, which minimizes (92).

Numerical illustration 2. The presented above method of minimization of the reactive and unbalanced currents with a compensator of reduced complexity is illustrated on an example of a system with distortion and asymmetry of the distribution voltage at the level of a few percents. Iit was assumed that $e_{\mathrm{S}} = 0.97e_{\mathrm{R}}$, $e_{\mathrm{T}} = 0.97e_{\mathrm{R}}$, $E_3 = E_5 = E_7 = 4\%$ of $E_1 = 100\mathrm{V}$. It was also assumed that the load is supplied from a weak source of short circuit