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Distribution function for plasma with RF heating from quasilinear Fokker-Planck equation

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Abstract

Auxiliary plasma heating by radio-frequency waves is a usual procedure in the modern tokamaks. In the case of ^{3}He minority heating is analysed the equilibrium distribution function for minority species and the factor modification of the Maxwellian form is plotted for specific parameters values as function of radial coordinate and normalized velocity. The energetic minority tail develops with the heating.

1 Introduction

The use of radio-frequency (rf) power in magnetic fusion devices has many important goals: heating plasma, current drive and profile control. Using rf power in the ion cyclotron resonance frequency (ICRF) to drive poloidal flow with radial velocity shear can stabilize microturbulence and improve plasma confinement.

In the present paper we consider a magnetically confined plasma in the presence of ion cyclotron resonant heating (ICRH) in the minority scheme. The knowledge of the distribution function is a key problem to solve both the heating efficiency of the plasma and the transport problem. The impact of ICRF heating on the distribution functions of various constituients are largely discussed in literature, see for example [1] - [7]. The plan of the paper is as follows:

In the section I, the frame and conditions of the simplified quasi-linear Fokker-Planck equation (QLFPE) is introduced. In the section II, the solution of the QLFPE is given in the condition of velocity space isotropy and the modification of the distribution function due to the presence of radio-frequency waves is evaluated. In the last section, for some specific values of parameters, the factor modification of the distribution function from Maxwellian is plotted and the results are discussed.

2 Quasilinear Fokker-Planck Equation

We consider a non-ohmic multi-component plasma heated at the ion cyclotron resonance for minority species, m. Let us consider the case of a low concentration of ions ^3He colliding with a thermal background plasma, composed by Deuterium and electrons. The ^3He minority is about $2\%-3\%$ of the density of the background plasma and it is heated by ion cyclotron resonant heating. We shall limit ourselves to analyze the so-called *simplified, quasi-linear, Fokker Planck equation* where it is taken into account only the contribution due to the perpendicular component of the electric field, $E_\perp^+$, which is concordant to the direction of rotation of the minority. We then neglect the contributions due to the perpendicular component of the electric field, $E_\perp^-$, which is discordant to the direction of rotation of the minority, and the one due to the parallel component of the electric field, $E_\parallel$. Under these conditions, in the velocity space, the long term evolution of the distribution function for the high frequency heated ions, is governed by the equation

$$\frac{\partial \mathcal{F}^m(\mathbf{y},t)}{\partial t} = -\nabla\cdot\mathbf{S}^m(\mathcal{F}^m) + P(\mathcal{F}^m) \tag{1}$$

where

$$\mathbf{S}^m(\mathcal{F}^m) = \bar{\mathbf{S}}^m_W(\mathcal{F}^m) + \sum_{\alpha=e,i}\bar{\mathbf{S}}^{m\alpha}_c(\mathcal{F}^m) \tag{2}$$

Suffices m and α [with $m = {}^3He$ and $\alpha = (e,D)$] distinguish the minority population and the species of the background plasma, respectively. The first term in Eq. (2) describes the quasi-linear diffusion due to the resonant wave particle interactions (RF contribution). The second term in Eq. (2) is due to the collisional operator. $P(\mathcal{F}^m)$ takes into account other auxiliary sources; in our analysis, we shall put $P(\mathcal{F}^m)=0$. The gradient operator, ∇, is defined as the row vector $\nabla \equiv (\partial_{v_x},\partial_{v_y},\partial_{v_z})$ in the velocity space, whilst $\nabla\cdot\bar{\bar{\mathbf{A}}}$ is the matrix multiplication between the gradient vector and the matrix $\bar{\bar{\mathbf{A}}}$. $\bar{\mathbf{S}}^{m\alpha}_c$ and the simplified quasi-linear term, $\bar{\mathbf{S}}^m_w$, can be written as

$$\begin{aligned}\bar{\mathbf{S}}^{m\alpha}_c(\mathcal{F}^m) &= -[\nabla\cdot(\bar{\bar{\mathbf{D}}}^{m\alpha(2)}_c\mathcal{F}^m)]^T + \bar{\mathbf{D}}^{m\alpha(1)}_c\mathcal{F}^m\\ \bar{\mathbf{S}}^m_W(\mathcal{F}^m) &= -[\nabla\cdot(\bar{\bar{\mathbf{D}}}^{m(2)}_W\mathcal{F}^m)]^T + \bar{\mathbf{D}}^{m(1)}_W\mathcal{F}^m\end{aligned} \tag{3}$$

where the expressions of the *matrix diffusion coefficients* ($\bar{\bar{\mathbf{D}}}^{m\alpha(2)}_c$ and $\bar{\bar{\mathbf{D}}}^{m(2)}_W$, for the collisional and the RF contributions, respectively) and the *drift vector coefficients* ($\bar{\mathbf{D}}^{m\alpha(1)}_c$ and $\bar{\mathbf{D}}^{m(1)}_W$, for the collisional and the RF contributions, respectively) have been introduced. T denotes the *transpose operation*. We shall provide the stationary solution, $\mathcal{F}^m$, of Eq. (1) in terms of the variables w defined as

$$w = \frac{\mathsf{v}}{\mathsf{v}_{thm}} \quad ; \quad \mathsf{v}_{thm} = \sqrt{\frac{2T_m}{m_m}} \tag{4}$$

where T_m , m_m , v denote the temperature, mass and velocity of the minority species, respectively. For the minority species ($\alpha = m$), introduce notation

$$\xi_\parallel = \frac{k_\parallel \mathsf{v}_{thm}}{\Omega_{cm}} \quad , \quad \xi_\perp = \frac{k_\perp \mathsf{v}_{thm}}{\Omega_{cm}} \quad , \quad \overline{\omega} = \frac{\omega}{\Omega_{cm}} \quad , \quad \Omega_{cm} = \frac{Z_m e B_0}{m_m c} \tag{5}$$

with Ω_{cm} denoting the *Larmor gyro-frequency* of the minority, Z_m the *charge number* of the minority and v_{thm} the *thermal velocity* of the minority species.

3 Solution of the quasilinear equation assuming isotropy

In the case of minority heating it is sufficient to regard the heated ions ($m = {}^3He$) as test particles colliding with a Maxwellian background plasma. Owing to the weak nonlinearity of the Fokker-Planck operator this is usually an acceptable approximation. If the applied power is not too large we can (as a first approximation) simplify the QLFPE further by neglecting the anisotropy which develops in the ion distribution function, $\mathcal{F}^m(w)$, which should satisfy the following one dimensional, steady-state, Fokker-Planck equation [7]

$$\frac{1}{\mathcal{F}^m}\frac{d\mathcal{F}^m}{dw} = -2\frac{\Psi_\tau(w)}{\Psi_c(w) + 2w\left\langle \overline{D}(\xi_\perp w)\right\rangle_{ql}}w \tag{6}$$

with the solution

$$\mathcal{F}^m(w) = \mathcal{F}_0^m \exp\left[-2\int_0^w \frac{\Psi_\tau(u)\ \ udu}{\Psi_c(u) + 2u\left\langle \overline{D}(\xi_\perp u)\right\rangle_{ql}}\right] \tag{7}$$

where

$$\Psi_c(w) \equiv \sum_\beta \frac{\nu^{m/\beta}}{\nu_m}\Psi\left(\frac{\mathsf{v}_{thm}}{v_{th\beta}}w\right) = \frac{\nu^{m/e}}{\nu_m}\Psi\left(\frac{\mathsf{v}_{thm}}{\mathsf{v}_{the}}w\right) + \frac{\nu^{m/D}}{\nu_m}\Psi\left(\frac{\mathsf{v}_{thm}}{\mathsf{v}_{thD}}w\right) \tag{8}$$

$$\Psi_\tau(w) \equiv \sum_\beta \frac{\nu^{m/\beta}}{\nu_m}\frac{T_m}{T_\beta}\Psi\left(\frac{\mathsf{v}_{thm}}{v_{th\beta}}w\right) = \frac{\nu^{m/e}}{\nu_m}\frac{T_m}{T_e}\Psi\left(\frac{\mathsf{v}_{thm}}{\mathsf{v}_{the}}w\right) + \frac{\nu^{m/D}}{\nu_m}\frac{T_m}{T_D}\Psi\left(\frac{\mathsf{v}_{thm}}{\mathsf{v}_{thD}}w\right) \tag{9}$$

$$\Psi(u) = \frac{1}{u^2}\left[\mathrm{erf}(u) - \frac{2}{\sqrt{\pi}}u\exp\left(-u^2\right)\right] \qquad \textit{Chandrasekhar function} \tag{10}$$

and $\mathcal{F}_0^m = \mathcal{F}^m(w=0)$ is the normalization integral. With simplified notations

$$A(w) \equiv \Psi\left(\frac{\mathsf{v}_{thm}}{\mathsf{v}_{the}}w\right) + \frac{\nu^{m/D}}{\nu^{m/e}}\Psi\left(\frac{\mathsf{v}_{thm}}{\mathsf{v}_{thD}}w\right) \tag{11}$$

$$B(w) \equiv \frac{T_m}{T_e}\Psi\left(\frac{\mathsf{v}_{thm}}{\mathsf{v}_{the}}w\right) + \frac{\nu^{m/D}}{\nu^{m/e}}\frac{T_m}{T_D}\Psi\left(\frac{\mathsf{v}_{thm}}{\mathsf{v}_{thD}}w\right) \tag{12}$$

the solution (7) becomes

$$\mathcal{F}^m(w) = \mathcal{F}_0^m \exp\left[-2\int_0^w \frac{B(u)\ \ udu}{A(u) + 2u\frac{\nu_m}{\nu^{m/e}}\left\langle \overline{D}(\xi_\perp u)\right\rangle_{ql}}\right] \tag{13}$$

Moreover, the isotropic part of the normalized quasilinear diffusion operator, see for example [6], reads as

$$\left\langle \overline{D}(\xi_\perp w)\right\rangle_{ql} = \frac{D_p}{2\nu_m \mathsf{v}_{thm}^2}\int_{-1}^{+1} d\lambda\ \left(1-\lambda^2\right)\ J_p^2\left(\xi_\perp w\sqrt{\left(1-\lambda^2\right)}\right) \tag{14}$$

with D_p given in terms of the 'initial' heating rate (when the distribution function is a Maxwellian)

$$D_p = \frac{P_{abs}^{lin}}{4n_m m_m \int_0^\infty w^3\ J_p^2(\xi_\perp w)\exp\left(-w^2\right)\ dw} \tag{15}$$

Here, $J_p(x)$ indicates the Bessel functions of the first kind, P_{abs}^{lin} is the power per unit volume supplied to the system, n_m is the number density of the minority species and λ the pitch angle, respectivelly. The coefficients D_p (with dimension of $\nu_m \mathsf{v}_{thm}^2$) are proportional to the power available per ion of the heated species.

With

$$\left\langle \overline{D}\left(\xi_{\perp} w\right)\right\rangle_{ql} \equiv \frac{\left\langle D\left(\xi_{\perp} w\right)\right\rangle_{ql}}{\nu_m \mathsf{v}_{thm}^2} \tag{16}$$

the equation (13) reads as

$$\mathcal{F}^m\left(w\right) = \mathcal{F}_0^m \exp\left[-2\int_0^w \frac{B\left(u\right)\ udu}{A\left(u\right)+2u\frac{\left\langle D\left(\xi_{\perp} u\right)\right\rangle_{ql}}{\mathsf{v}_{thm}^2 \nu^{m/e}}}\right] \tag{17}$$

with $\mathcal{F}_0^m = \mathcal{F}_0^m(r\ ; w = 0)$. When $\left\langle D\left(\xi_{\perp} w\right)\right\rangle_{ql} \equiv 0$ and all species have the same temperature, $\mathcal{F}^m\left(w\right)$ reduces to unperturbed Maxwellian, $F_M^m(w)$,

$$\mathcal{F}_M^m(w) = \mathcal{F}_0^m \exp(-w^2) \tag{18}$$

The equilibrium distribution function $\mathcal{F}^m(w)$ corresponding to the minority species heated by rf heating will be assumed of the form

$$\mathcal{F}^m(w) = \chi_1^m\left(\mathbf{r}\right)\mathcal{F}_M^m(w) \tag{19}$$

and so

$$\chi_1^m \equiv \exp\left\{w^2 - 2\int_0^w \frac{B\left(u\right)\ udu}{A\left(u\right)+2u\frac{\left\langle D\left(\xi_{\perp} u\right)\right\rangle_{ql}}{\mathsf{v}_{thm}^2 \nu^{m/e}}}\right\} \tag{20}$$

To lowest order in the thermal Larmor radius, for minority heating ($p = 0$),

$$D_0 = \frac{P_{abs}^{lin}}{4n_m m_m \int_0^{\infty} w^3\ J_0^2\left(\xi_{\perp} w\right)\exp\left(-w^2\right)\ dw} \tag{21}$$

It is easily checked that in the limit $\xi_{\perp} w \ll 1$, we find

$$\int_0^{\infty} w^3\ J_0^2\left(\xi_{\perp} w\right)\exp\left(-w^2\right)\ dw \rightarrow \frac{1}{2} \quad , \quad \text{for} \quad \xi_{\perp} w \ll 1$$

and

$$D_0 = \frac{P_{abs}^{lin}}{2n_m m_m} \qquad \text{for} \quad p = 0 \ \ \text{and} \ \ \xi_{\perp} w \ll 1 \tag{22}$$

As can be seen from (11) and (12), the collisions between minority species (seen as a test particle) and the background particles (electrons and majority ion species) are taking into account through the quantities $A\left(r,w\right)$ and $B\left(r,w\right)$.

With

$$\nu_m = \nu^{m/D} + \nu^{m/e} \approx \nu^{m/D} \tag{23}$$

we have

$$\chi_1^m(r,w) \equiv \exp\left\{ w^2 - 2\int_0^w \frac{B(r,u)\ udu}{A(r,u) + 2u\frac{\langle D(\xi_\perp u\ ;\ r)\rangle_{ql}}{\mathsf{v}_{thm}^2 \nu^{m/e}}} \right\} \tag{24}$$

With collision frequencies, see for example [8],

$$\nu^{a/b} = 4\pi \frac{n_b e_a^2 e_b^2\ \ln \Lambda}{m_a^2 \mathsf{v}_{Ta}^3} \tag{25}$$

we obtaine

$$\nu^{m/e} = 4\pi \frac{n_e Z_m^2 e^4\ \ln\Lambda}{m_m^2 \mathsf{v}_{Tm}^3} \quad , \quad \nu^{m\backslash D} = 4\pi \frac{n_D e_m^2 e_D^2\ \ln\Lambda}{m_m^2 \mathsf{v}_{Tm}^3} \quad , \quad \frac{\nu^{m/D}}{\nu^{m/e}} = Z_D^2 \frac{n_D}{n_e}$$

$$A(r,w) = \Psi\left(\frac{\mathsf{v}_{thm}}{\mathsf{v}_{the}} w\right) + Z_D^2 \frac{n_D}{n_e} \Psi\left(\frac{\mathsf{v}_{thm}}{\mathsf{v}_{thD}} w\right) \tag{26}$$

$$B(r,w) = \frac{T_m}{T_e} \Psi\left(\frac{\mathsf{v}_{thm}}{\mathsf{v}_{the}} w\right) + Z_D^2 \frac{n_D}{n_e} \frac{T_m}{T_D} \Psi\left(\frac{\mathsf{v}_{thm}}{\mathsf{v}_{thD}} w\right) \tag{27}$$

4 Conclusions and discussions

In the following we take particular values of the parameters in order to plot numerically the function $\chi_1^m(w)$. The perpendicular wave number $k_\perp$ is assumed as $k_\perp \approx 1/\rho_{Lm}$ and so $\xi_\perp = k_\perp \mathsf{v}_{thm}/\Omega_{cm} = 1$. The initial density radial profiles for electron and Deuterium majority ion species are given in Fig.1 and Fig.2, respectively.

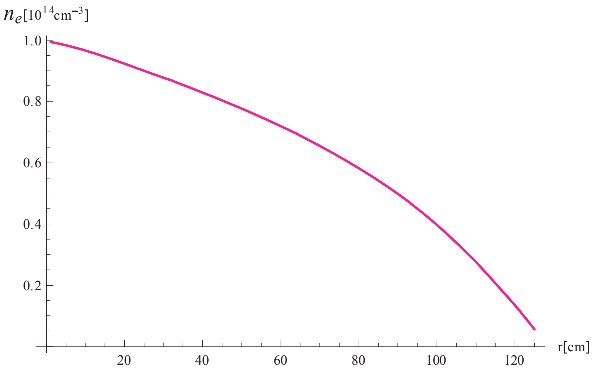

Fig.1 Electron density radial profile.

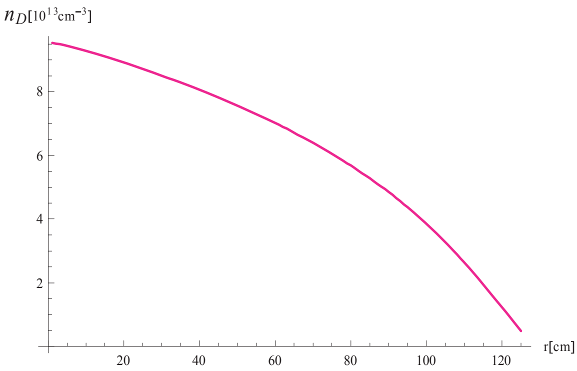

Fig.2 Majority ion species (Deuterium) density radial profile.

Also the initial temperature (before heating) radial profiles for electron (Fig.3) and ^3He minority species (Fig.4) are given.

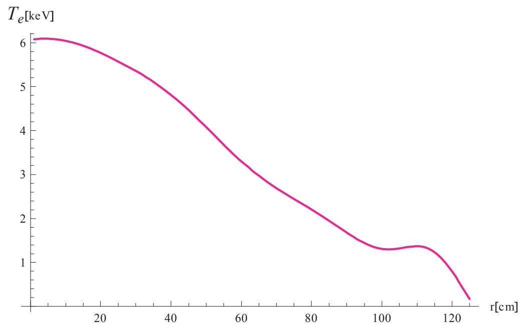

Fig.3 Electron temperature radial profile.

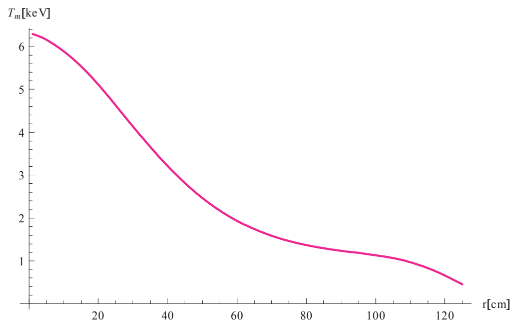

Fig.4 Minority ion species (^{3}He) temperature radial profile.

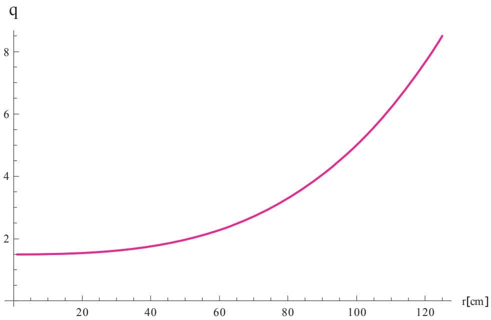

Fig.5 Safety factor q

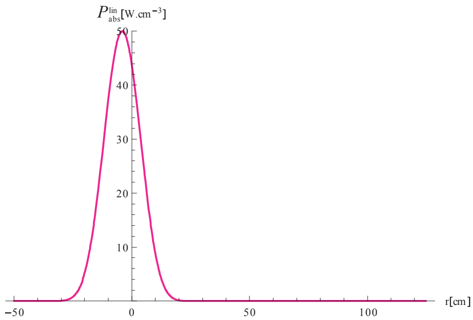

Fig.6 Power density absorption P_{abs}^{lin} radial profile.

If the ion velocity distribution is Maxwellian the absorbed rf power density, $\langle P \rangle$, averaged on the flux surface has a Gaussian profile within the resonance layer,

$$P_{abs}^{lin} \approx \langle P \rangle = P_0 \exp\left[-\frac{(r - a\cos\theta_{res})^2}{2\,(\triangle X)^2} \right]$$

where θ_{res} is the poloidal angle corresponding to the central vertical axes of the resonance layer, a the minor radius and

$$\triangle X \simeq R_0 \frac{k_{||} v_{thi}}{n \Omega_{ci,0}} \tag{28}$$

We assume the resonance absorption layer is centered near the magnetic axes on the higher magnetic field side ($\theta_{res} = 0.51\pi$) with a maximum value $P_{abs}^{lin} = 50\ W/cm^3$ of the power density absorption (see Fig.6).

In this conditions we plot numerically the function $\chi_1^m\,(r, w)$ for two values of the normalized velocity ($w = 0.7$ and $w = 1$) of the minority species - see Fig7.

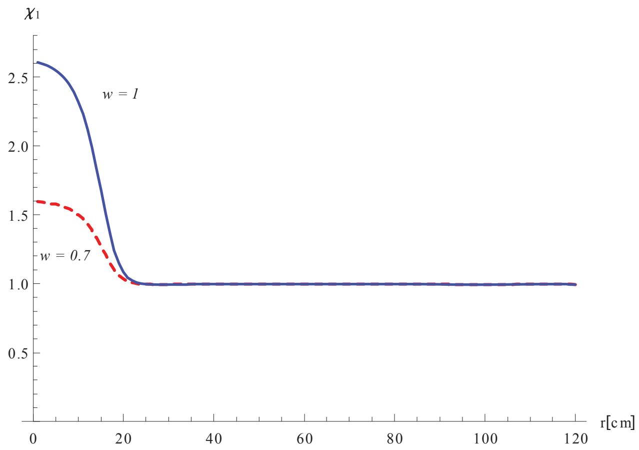

Fig.7 The deformation factor $\chi_1^m(r, w)$ of the Maxwellian distribution as function of r for two fixed values of w. (Here $\xi_\perp = 1$).

The distribution function for the heated minority species is modified in the region of the power absorption and is more pronounced with higher w. We note that the validity of equation (24) is restrictioned by the condition $\xi_\perp w \leq 1$. So, for lower values of $k_\perp$ (equivalently $\xi_\perp$) we can study for higher values of w.

A value of $\chi_1^m(r) > 1$ (for given w) leads to a higher value of the distribution function $\mathcal{F}^m(r)$ and so to a higher density n_m and density gradient. This situation can not last because of diffusion.

For a position well inside the resonance layer (e.g $r = a/6$) the factor $\chi_1^m(w)$ increase monotonically when w is incresing (see Fig.8). An interesting situation appear in the region with low absoption (at border of the resonance layer) where the factor $\chi_1^m(w)$ has a non-monotonically variation with a minimum at $w \approx 1$ and $\chi_1^m < 1$ for this spatial region and $\xi_\perp w \lesssim 3/4$ - see Fig.9. This also show that the heating develops the high energetic minority tail.

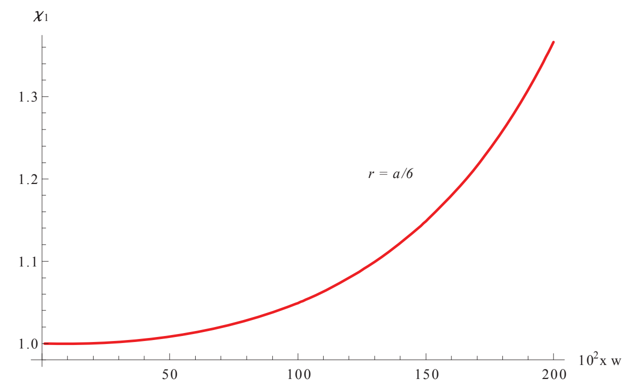

Fig.8 The factor $\chi_1^m(w)$ for $r = a/6$ and $\xi_\perp = 1/2$.