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Qualitative Study of Differential Equations, Geometrical and Dynamical Aspects of Some Mechanical Systems, Numerical Treatment, and Applications

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Preface

This monograph generally represents a study on differential equations, starting from their qualitative overview, by emphasizing both geometrical and dynamical aspects, at last but not at least accentuating their various applications, through concrete models from Mechanics, Medicine, and not only. The subject of the work is of high topical, having relevant theoretical and applicative results.

The authors are members of a research group on nonlinear analysis, differential geometry, and numerical analysis, of the Department of Applied Mathematics of the University of Craiova.

The Chapters of this book structure the main authors' scientific interests within the last years, as well as their latest results.

Therefore, the work is organized in 11 Chapters and starts with a qualitative study of differential equations, presenting in Chapter 1, written by Aurel Diamandescu, wherein necessary and sufficient conditions for $\Psi -$ conditional asymptotic stability of the trivial solution of linear or nonlinear Lyapunov matrix differential equations, are presented.

In Chapter 2, written by Cristian Vladimirescu, we prove theorems of the existence of solutions to some ODE's of second order, defined on the half-line as well as on the whole real line, and having finite limits on the boundary. The proof techniques are mainly based on the application of fixed point theorems and differential inequalities.

In Chapter 3, written by Mihaela Racilă and Jean-Marie Crolet, there are presented a few numerical methods with applications in medicine. It concerns a mathematical model of human cortical bone, based on the homogenization theory in a piezoelectric framework and numerical simulations by the finite element method. This model allows the study of the mechanical behavior of the heterogeneous structure of cortical bone, knowing the properties of its basics components and its architectural configuration.

The variable exponent spaces are the subject of an intense investigation lately, mainly due to their applicability to electrorheological fluids, thermorheological fluids, elastic materials, image restoration, mathematical biology, dielectric breakdown, electrical resistivity and polycrystal plasticity. Thus, besides some history notes, we present in Chapter 4, written by Maria-Magdalena Boureanu, some basic traits of the isotropic and anisotropic spaces with variable exponents. As for the usage of these properties in the study of the boundary value problems, we focus on the anisotropic problems with variable exponents.

In Chapter 5, written by Cristian-Paul Dăneţ, we define several types of functions on the solution to a class of linear elliptic equations of even order including a class of fourth order equations arising in plate theory. We establish that these functions satisfy a classical maximum principle. As a consequence we obtain uniqueness results and bounds on various quantities of interest in Mechanics.

Chapter 6, written by George Popescu, emphasizes how commutativity influences inequalities on $C^{\star}$-algebras, then defines "completely positive" inner products as an alternate tool in producing "non-commutative" inequalities, in particular a non-commutative version of Schwarz inequality.

In Chapter 7, written by Marcela Popescu and Paul Popescu, we deal with some problems in geometric foliations theory, giving some necessary and sufficient conditions that a foliation be a Riemannian one. These conditions involve the presence of some Lagrangians or Hamiltonians on some geometric transverse spaces (jet or accelerations). A new form of nonholonomic Chetaev conditions, according to some nonlinear constraints adapted to a foliation, are also given.

The problem of finding symmetries and conservation laws for different dynamical systems generated by systems of ordinary differential equations (SODE) or by systems of partial differential equations (SOPDE) is still of a great interest. There is a very well-known way to obtain conservation laws for a system of differential equations given by a variational principle: the use of the Noether Theorem which associates to every symmetry a conservation law and conversely. However, there is a method introduced by G.L. Jones and M. Crâşmăreanu by which new kinds of conservation laws can be obtained without applying a theorem of Noether type, only using symmetries and pseudosymmetries. In Chapter 8, written by Florian Munteanu, we will extend this method to the presymplectic case and also to the Lagrangian and Hamiltonian k-symplectic formalism.

A system of second order differential equations (SODE or spray) is a vector field (locally defined) on the tangent bundle TM whose integral curves are the solutions of the Euler-Lagrange equations. In Chapter 9, written by Florian Munteanu, we extend the study of the relationship between sprays (SODEs) and nonlinear connections from the tangent bundle TM of a manifold M to the k-tangent bundle T^kM, taking into account that a k-spray on T^kM represents a system of ordinary differential equations of $k+1$ order, whose coefficients are invariant under the coordinates changes. For the other type of high order tangent bundle, that means the tangent bundle of k^1-velocities of a manifold M, T^1_kM, we will prove that any system of second order partial differential equations (SOPDEs) determines at least two nonlinear connections and conversely.

Chapter 10, written by Romulus Militaru and Florian Munteanu, focuses on the interplay between dynamical systems geometrical theory and computational calculus of dynamical systems. The viewpoint is geometric and we also compute and characterize objects of dynamical significance, in order to understanding the mathematical properties observed in numerical computation for dynamical models arising in mathematics, science and engineering. Starting with the analysis of the conservation laws of Hamiltonian dynamical system associated to the isotropic harmonic oscillator for one dimensional case and two dimensional case, we continue with a geometrical and numerical study of the main sizes of the mathematical models of the multispecies interactions which are important in determining long-time dynamics, based on the application of various notions from the theory of dynamical systems to the numerical approximation of initial value problems over long-time intervals.

In Chapter 11, written by Adela Ionescu, the work from the past recent years, on turbulent mixing mathematical model, is continued. The comparisons of deformation (in length and surface) efficiency of 3D versus 2D case brought a very rich panel of random distributed events. An important partial conclusion was that the mixing, and especially the turbulent mixing, is introduced at irrational values of length and surface versors. There is used a modern mathematical soft, MAPLE11, which has a lot of performing computational appliances. A logistic type term is tested for the 3D mixing mathematical model, and the trajectories behavior for this new model is analyzed.

A series of numerical simulations incline to strengthen the presented studies and to "convince" the rightful necessity of the join between the qualitative study of differential equations and the numerical simulations. The work thus purposeful, wishes to be an useful matter both for the acknowledged researchers and the students from the adjacent Master's specializations, PhD students, and postdoctoral students, who wish to focus their research to this field found at the intersection of the pure with the applied Mathematics.

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CHAPTER 1

On the $\Psi-$ Conditional Asymptotic Stability of Nonlinear Lyapunov Matrix Differential Equations

1.1. Introduction

The Lyapunov matrix differential equations occur in many branches of control theory such as optimal control and stability analysis.

Recent works for $\Psi-$ boundedness, $\Psi-$ stability, $\Psi-$ instability, controllability, dichotomy and conditioning for Lyapunov matrix differential equations have been given in many papers. See, for example, [**5**]-[**14**] and the references cited therein.

The purpose of present paper is to prove (necessary and) sufficient conditions for $\Psi-$ conditional asymptotic stability of trivial solution of the nonlinear Lyapunov matrix differential equation

$$Z' = A(t)Z + ZB(t) + F(t, Z) \tag{1.1}$$

and the linear Lyapunov matrix differential equation

$$Z' = [A(t) + A_1(t)]Z + Z[B(t) + B_1(t)], \tag{1.2}$$

which can be seen as a perturbed equations of the linear equation

$$Z' = A(t)Z + ZB(t). \tag{1.3}$$

We investigate conditions on the fundamental matrices of the equations

$$Z' = A(t)Z, \tag{1.4}$$

$$Z' = ZB(t) \tag{1.5}$$

and on the functions A_1, B_1 and F under which the trivial solutions of the equations (1.1)-(1.3) are $\Psi-$ conditionally asymptotically stable on $\mathbb{R}_+$. Here, Ψ is a matrix function whose introduction permits us obtaining a mixed asymptotic behavior for the components of solutions.

The main tool used in this paper is the technique of Kronecker product of matrices, which has been successfully applied in various fields of matrix theory, group theory and particle physics. See, for example, the above cited papers and the references cited therein.

1.2. Preliminaries

In this section we present some basic definitions, notations, hypotheses and results which are useful later on.

Let $\mathbb{R}^d$ be the Euclidean d-dimensional space. For

$$x = (x_1, x_2, x_3, ..., x_d)^T \in \mathbb{R}^d,$$

let $\| x \| = \max\{| x_1|, | x_2|, | x_3|, ..., | x_d |\}$ be the norm of x (^{T} denotes transpose).

Let $\mathbb{M}_{d\times d}$ be the linear space of all $d \times d$ real valued matrices.

For $A = (a_{ij}) \in \mathbb{M}_{d\times d}$, we define the norm $| A |$ by $| A | = \sup\limits_{\|x\|\leq 1} \| Ax \|$. It is well-known that $| A | = \max\limits_{1\leq i\leq d}\{\sum\limits_{j=1}^{d} | a_{ij} |\}$.

By a solution of the equation (1.1) we mean a continuous differentiable $d \times d$ matrix function satisfying the equation (1.1) for all $t \in \mathbb{R}_+$.

In equation (1.3), we assume that A and B are continuous $d \times d$ matrices on $\mathbb{R}_+ = [0, \infty)$. It is well-known that continuity of A and B ensure the existence and uniqueness on $\mathbb{R}_+$ of a solution of (1.3) passing through any given point $(t_0, Z_0) \in \mathbb{R}_+ \times \mathbb{M}_{d\times d}$.

In addition, in equation (1.1), we assume that $F : \mathbb{R}_+ \times \mathbb{M}_{d\times d} \longrightarrow \mathbb{M}_{d\times d}$ is continuous such that $F(t, O_d) = O_d$ (null matrix of order $d \times d$).

It is well-known that these conditions ensure the local existence of a solution passing through any given point $(t_0, Z_0) \in \mathbb{R}_+ \times \mathbb{M}_{d\times d}$, but it not guarantee that the solution is unique or that it can be continued for large values of t.

Let $\Psi_i : \mathbb{R}_+ \longrightarrow (0, \infty)$, $i = 1, 2, ..., n$, be continuous functions and

$$\Psi = \text{diag } [\Psi_1, \Psi_2, \cdots \Psi_n].$$

Definition 1.1. ([**3**], [**7**]). *The solution z(t) of the differential equation $z' = f(t, z)$ (where $z \in \mathbb{R}^d$ and f is a continuous d vector function) is said to be $\Psi-$ stable on $\mathbb{R}_+$ if for every $\varepsilon > 0$ and every $t_0 \in R_+$, there exists a $\delta = \delta(\varepsilon, t_0) > 0$ such that, any solution $\widetilde{z}(t)$ of the equation which satisfies the inequality $\| \Psi(t_0)(\widetilde{z}(t_0) - z(t_0)) \| < \delta$, exists and satisfies the inequality $\| \Psi(t)(\widetilde{z}(t) - z(t)) \| < \varepsilon$ for all $t \geq t_0$.*

Otherwise, is said that the solution z(t) is $\Psi-$ unstable on $\mathbb{R}_+$.